Modeling Double–Diffusive Processes in Ocean and Stars

Friedrich Kupka[1], **Martin Losch**[2], **Florian Zaussinger**[1] **and Thomas Zweigle**[2]

[1]Max–Planck–Institut für Astrophysik, Garching; E–Mail: fk@mpa-garching.mpg.de, fzaussinger@mpa-garching.mpg.de

[2]Alfred–Wegener–Institut, Bremerhaven; E–Mail: mlosch@awi.de, tzweigle@awi.de

Double–Diffusion in the Ocean

1 Introduction

On the whole, the ocean is stably stratified with lighter water overlying denser water. Because the ocean's density is determined by both temperature and salinity, double diffusive processes can, in certain situations, erode the statically stable stratification. There are two main types of double-diffusive processes in the ocean:

1. Saltfingering: with warm and salty water over cold and less salty water, the vertical salinity gradient destabilizes and the temperature gradient stabilizes the water column (e.g. Tyrrhenian Sea, Caribbean Sea)
2. Semiconvection: with cold and less salty water over warm and salty water, the vertical salinity gradient stabilizes and the temperature gradient destabilizes the water column (e.g. Arctic, melting sea ice)

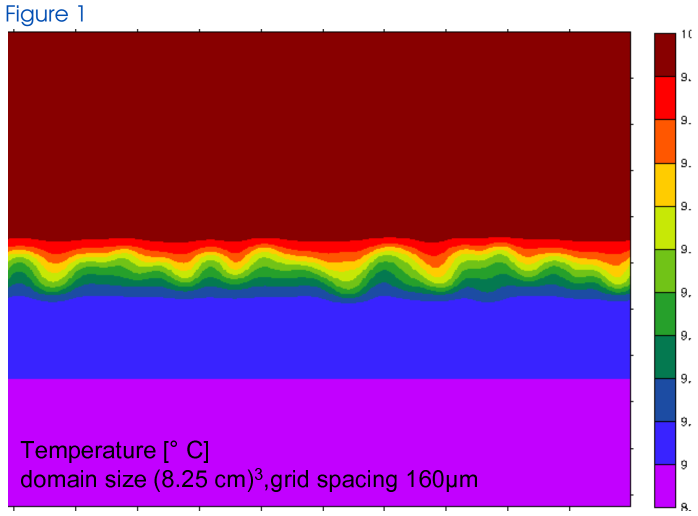
Thus, double-diffusive processes can increase the vertical transport of material where otherwise there would be no material exchange. Double–diffusion may have an observable effect on large scale circulation patterns in the ocean [1], [2].

2 3D–Simulations of Saltfingers with "DNS"

We simulate double-diffusive processes using the finite-volume ocean model MITgcm [3]. In the first part of our work, **D**irect **N**umerical **S**imulations of 2D and 3D problems provide estimates of heat and salinity fluxes as well as diffusive fluxes from heat and salinity. Resolving $\tau \cdot \sigma \approx 700$ represents a challenge.

The molecular diffusivities for heat (κ_T) and salt (κ_S) are $1.5 * 10^{-7}\, m^2/sec$ and $1.5 * 10^{-9}\, m^2/sec$.

Figure 1



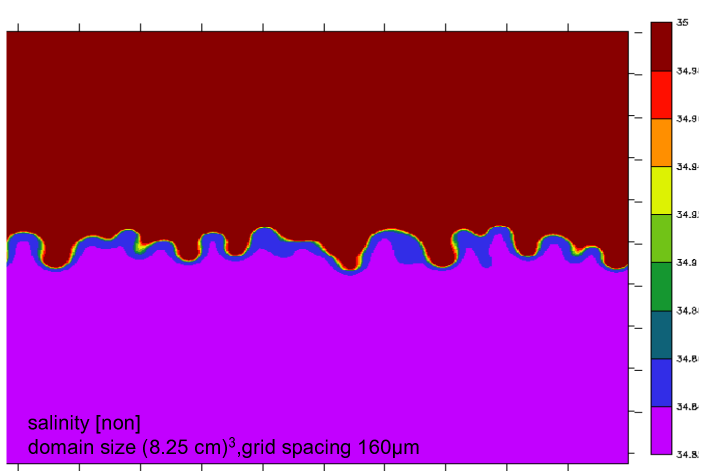


Figure 1 shows a vertical section of a 3D saltfinger simulation. In the beginning of the simulation, only molecular diffusion drives the system. Because $\kappa_T \gg \kappa_S$, saltfingers form, which lead to upward and downward fluxes of salinity. The net flux is directed downwards.

3 First Results

With time, the influence of diffusion decreases and advection of temperature and salinity dominates (see also Figure 3). The domain will be well mixed by advective processes in the next 70sec of the integration.

Figure 2 shows the horizontal mean of temperature and salinity at 40sec, 50sec and 65sec. After the temperature step became smoother by diffusion – and so the buoyancy near the interface decreases – the denser, salty water in the upper layer sinks down by gravity and forms drops.

These are the Saltfingers which are observable in Figure 3 by comparing the different fluxes and Figure 1 where saltfingers arise.

Figure 2

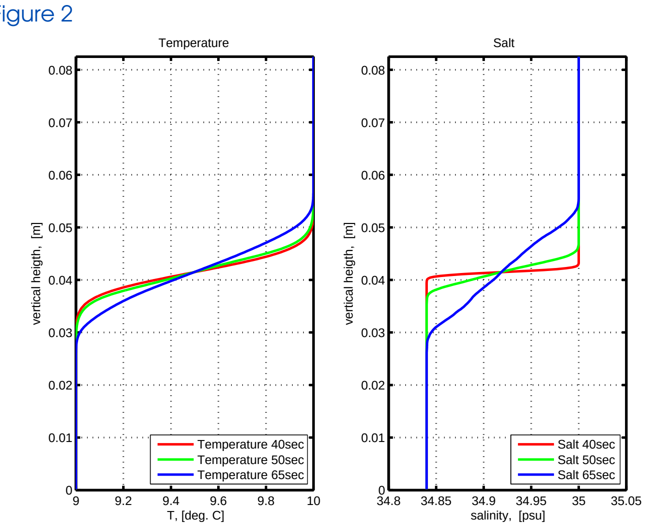
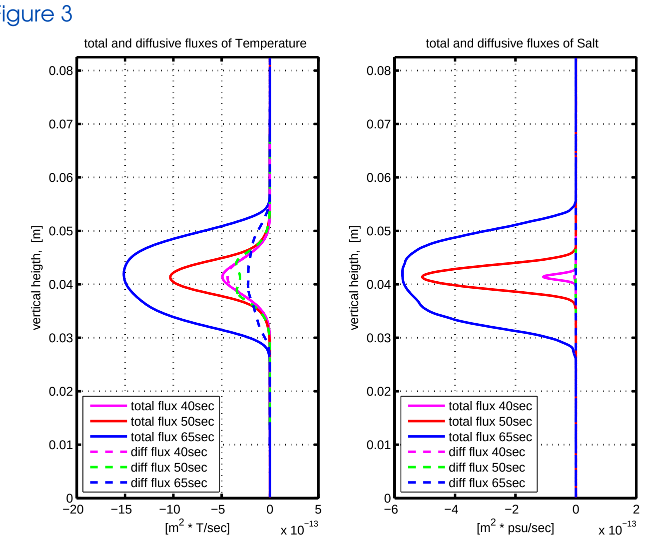


Figure 3



While the diffusive flux of salinity is three orders of magnitude smaller than that of heat and the total fluxes of heat and salinity are of similar magnitude, the total flux is of salinity three orders of magnitude bigger than the diffusive flux of salinity (not seen in this figure). Thus, by triggering instability and subsequently turbulent processes, saltfingering enhances the net salinity flux.

4 Next Steps

- Simulations with more than 1024^3 grid points
- find values for κ_T/κ_S for coarse models depending on **DNS** fluxes
- use parameterisations obtained from the **"DNS"** part to improve **LES** and existing ocean models
- compare to stellar conditions where Lewis and Prantl numbers are similar, but spatial and temporal scales are much larger.

Furthermore, we estimate the contribution of double–diffusion to vertical transport processes.

References

[1] M. Merryfield, Origin of Thermohaline Staircases, J. Phys. Oceanograpy, Vol 30, (1999)

[2] T. Radko, A Mechanism for layer formation in a double–diffusive fluid, J- Fluid Mech. vol. 597, (2003)

[3] **M**assachusetts **I**nstitute of **T**echnology **G**eneral **C**irculation **M**odel, http://mitgcm.org

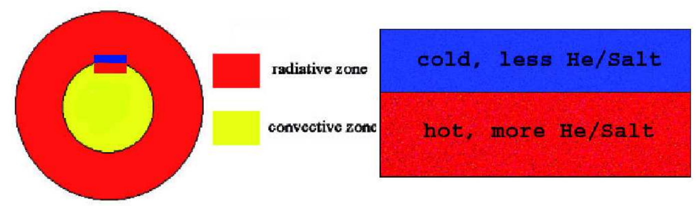
Semiconvection in massive stars

5 Introduction

Double diffusion occurs when two different active scalars exist, eg. T and μ, which have very different kinematic diffusivities. Diffusive convection is characterized by a stable T (temperature) gradient and an unstable μ (molecular weight) gradient. In stars, diffusive convection is called semiconvection under the assumption, that the temperature gradient destabilizes and the molecular weight gradient stabilizes a certain region. Semiconvection plays a fundamental role in understanding the evolution of massive stars. It may lead to structural modifications outside the convective core that have significant effects on later phases of stellar evolution. Cold He-poor plasma is stratified above hot He-rich material, which yields the existence of a destabilizing temperature gradient ∇T and a stabilizing molecular weigth gradient $\nabla \mu$.

Main goals:

1. Calibration of models of this physical process for stellar structure and evolution models.
2. Validation of the appropriate physical scenario of semiconvection.



6 Low mach number approximation

Fluid speeds in semiconvection zones are of the order of 10^{-3} Mach. Therefore a low mach number approximation, based on Navier-Stokes equations, was derived to filter out acoustic waves and to increase the size of the timesteps. A constraint on the velocity field allows an infinitely fast acoustic equilibration, which is mathematically expressed by an additional Poisson equation. Thus, we posit the existence of a background state with p, ρ, T, satisfying both the equation of state and hydrostatic equilibrium. Depending on the temperature gradient and the helium gradient two different cases (Ledoux stable/Ledoux unstable) can be destinguished.

7 Criterions for semiconvection

$\nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$ molecular weight gradient

$\nabla_{ad} := (\frac{\partial \ln T}{\partial \ln P})_{ad}$ adiabatic gradient

$\nabla := \frac{\partial \ln T}{\partial \ln P}$ temperature gradient

$R_\mu = \frac{\nabla_\mu}{\nabla - \nabla_{ad}}$ stability parameter

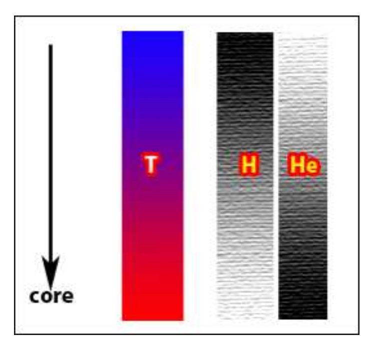
$N^2 = g H_p^{-1}(\nabla_\mu - (\nabla - \nabla_{ad}))$ Brunt-Väisälä frequency

semiconvection, if:	$\nabla - \nabla_{ad} > 0$	$\nabla_\mu > 0$	$R_\mu > 0$
Ledoux stable:	$N^2 > 0$	$\nabla_\mu > \nabla - \nabla_{ad}$	$R_\mu > 1$
Ledoux unstable:	$N^2 < 0$	$\nabla_\mu < \nabla - \nabla_{ad}$	$R_\mu < 1$

8 Simulations

First basic 2D simulations have been done with the software tool ANTARES (Advanced Numerical Tool for Astrophysical RESearch). A low mach number solver for semiconvection was implemented to simulate chemical mixing in the interior of a 25 solar mass star. Simulating the entire star is not feasible, so a "box-in-the-star" model is choosen. Typical domains are in the order of $2Mm$ with a resolution of $20km$ in each direction. The main goal is to understand the nature of semiconvection by simulating its behavior numerically.

Figure 4

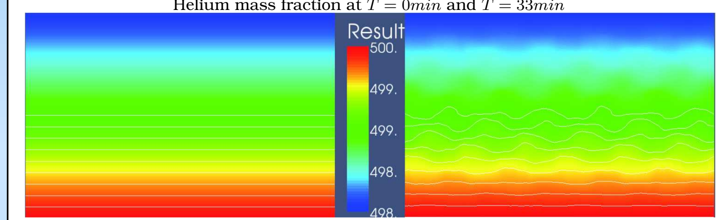


Typical values for temperature and density are of the order of $20MioK$ and $0.6g/cm^3$. The Prandtl number is in the range of $\ll 10^{-6}$, while $\frac{Le}{Pr} \approx \frac{1}{100}$ and $\kappa_{He} \ll \nu \ll \kappa_T$.

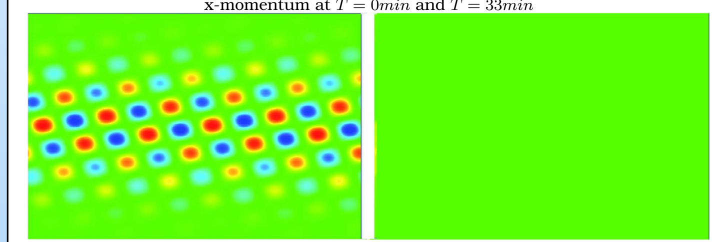
9 Simulations

Ledoux stable case:
Grid: 160 vertical x 240 horizontal $\cong$ 1500 km x 2250 km $\cong \triangle x = 9.43km$
$\triangle t = 5 * 10^{-3} scrt\ Pr = 0.05,\ Le = 0.04$ total simulation time: 33min

Helium mass fraction at $T = 0min$ and $T = 33min$



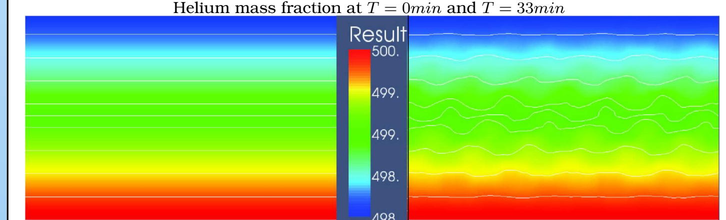
x-momentum at $T = 0min$ and $T = 33min$



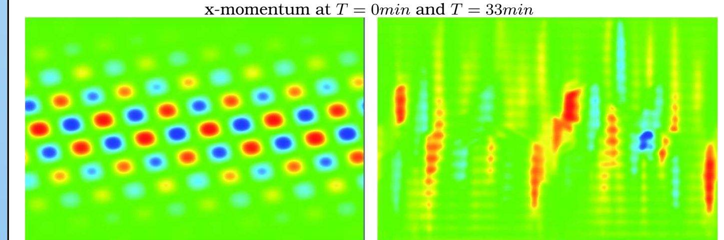
While in the Ledoux stable case no momentum fluctuations are observed after 30 min, the unstable scenario appears to continue mixing.

Ledoux unstable case:
Grid: 160 vertical x 240 horizontal $\cong$ 1500 km x 2250 km $\cong \triangle x = 9.43km$
$\triangle t = 5 * 10^{-3} scrt\ Pr = 0.25,\ Le = 0.0175$ total simulation time: 33min

Helium mass fraction at $T = 0min$ and $T = 33min$



x-momentum at $T = 0min$ and $T = 33min$



The inital perturbation (sin function) in both cases is applied to momentum. Due to the fact that appropriate initial perturbations can reduce relaxation time and avoid artefacts, it is important to choose them in a correct physical manner. Therefore a random perturbation on the momentum field was coded. Tests with these new initial conditions will follow.

10 Outlook

Phase I: Current 2D tests have been done up to 33min, which have to be extended up to 2days ($\simeq$ 100.000 sound crossing times) to analyze all mixing effects. Massively high performance computing using also OpenMP is under preparation, which will provide more flexibility when used together with MPI parallization. In the next stage of Phase I the reproduction of existing semiconvection simulations (Merryfield, Biello) in a more realistic stellar parameter space is planned.

Phase II: 3D simulations with a resolution of 1024 grid points in each direction in a $2Mm^3$ domain are planned. An application to exoplanet atmospheres is also considered as well as a local gridrefinement zone.

11 Collaborations

1. Universität Wien (ACORE group): H. Muthsam
2. Universität Hannover (PALM group): S. Raasch

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