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Correction to "Present-day uplift patterns over Greenland from a coupled ice-sheet/visco-elastic bedrock model"

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In the paper "Present-day uplift patterns over Greenland from a coupled ice-sheet/visco-elastic bedrock model" by E. Le Meur and P. Huybrechts, Geophysical Research Letters, 25 [21], 3951-3954, negative signs were omitted from several equations in the print edition. The affected sections are included below in their entirety:

Coupling of the two Models

The coupling consists first in forcing the bedrock model with the loading from the ice model. With these loading data, the bedrock model computes the corresponding new bedrock topography which is then reinserted in the ice model so that the effects of bedrock changes on the ice dynamics can be fully accounted for. The viscous part of the current bedrock response is by more than 99 % determined by the loading history over the last 30 kyrs. The discrete time-integration over this period of the viscous Love numbers arising from the impulse model (a viscous displacement amplitude $h_n^j(a)$ and the inverse of a decaying period $1/s_n^j$ for each of the 4 main viscous modes) with a 100 yr (Δt) time-step therefore gives $N_m - 1$ elementary contributions $G_{n,k}(a)$ (with $N_m = 30000yrs/\Delta t = 300$). So defined, $G_{n,k}(a) = \sum_{j=1}^{4} \frac{h_n^j(a)}{s_n^j}[e^{s_n^j(N_m-k)\Delta t} - e^{s_n^j(N_m-k-1)\Delta t}]$ represents the radial viscous displacement in response to the unit load that prevailed between $(N_m - k - 1)\Delta t$ and $(N_m - k)\Delta t$ before the current time t. Each of these contributions is then weighted by the corresponding load difference between the time considered and the reference state $L_{i_1,j_1}(k)$ at the (i_1, j_1) point before being summed over time from $k = 1$ to $k = N_m - 1$ as shown in eq. 1. To incorporate the regional character of the bedrock response, the grid over which the loads are calculated is extended beyond the 83 x 141 ice-sheet model grid by a radius of influence set to 1000 km. As a consequence, not only the ice loading (ice density times the local ice thickness H_{i_1,j_1}), but also the water loading (sea-water density times the water column) are considered, thereby enabling the incorporation of the effects of sea level changes. The elastic contribution only implies one elastic surface Love number amplitude $h_n^E(a)$ (asymptotic limit of the general model solution when inverting the Laplace transform, see *Wu and Peltier*, [1982]) and the current state of loading $L_{i_1,j_1}(N_m)$.

The spatial convolution for each (i,j) point of the numerical grid incorporates the contribution to deformation of all the neighbouring (i_1, j_1) points within the radius of influence $D_{i,j}$. The colatitudinal dependence of the total bedrock response (viscous + elastic) is obtained by summing a Legendre series up to an harmonic cut off set at $N_{max} = 200$, in which $P_n(\cos\theta_{i,i_1,j,j_1})$ represents the Legendre polynomial and θ_{i,i_1,j,j_1} the angle at the surface of the Earth between (i_1, j_1) and (i, j)). Since each grid point has its own time history, the time and space convolutions cannot be split. This gives for the surface radial displacement R at node (i,j) and time t :

$$R_{ij}(t) = \sum_{i_1,j_1 \in D_{i,j}} \Big[\sum_{n=0}^{200} \frac{a}{M_e}\Big(\sum_{k=1}^{N_m-1} L_{i_1,j_1}(k)\, G_{n,k}(a) + L_{i_1,j_1}(N_m)\, h_n^E(a)\Big) P_n(\cos\theta_{i,i_1,j,j_1})\Big] \Delta x \Delta y \qquad (1)$$

in which a/M_e is the Earth's radius/mass ratio resulting from the dimensionless Love numbers [*Wu and Peltier,* 1982] and Δx, Δy are the x and y grid spacings (20 km for both).

Results of the Simulation

The simultaneous evolution of total ice volume and mean bedrock height for the last 125000 are displayed in Figure 1. They appear to be well correlated during the slow built-up phase, though the bedrock uplift displays a time lag of a few thousand years in response to the recent faster deglaciation. This non-linearity is a direct consequence of the discrepancy between the long time constant of the viscous mantle and the fast evolving load of a decaying ice sheet. As the Earth viscous response is a function of the past loading events, an ongoing future evolution of the mean bedrock elevation occurs despite an almost constant load (rightmost part of Figure 1).

Figure 2 shows a clear correspondence between the deglaciated areas since the Last Glacial Maximum and the total corresponding uplift. The similarity is affected by the rigid behavior of the lithosphere, which acts as a low-pass filter and makes the response regional instead of local. Therefore, the result is a smoothed imprint of the loading change pattern which consists of both a discontinuous peripheral deglaciation belt, especially in the southwest and northeast of Greenland, and a central thickening of the ice sheet.

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Calculating the uplift rate requires a different version of eq. 1 in which the loading function L is replaced by its time derivative in the elastic part and by the finite difference formula $\frac{(L(k+1)-L(k)}{\Delta t}$ for the viscous part. The definition of the time period to calculate the present-day ice thickness change (dL/dt) remains however ambiguous. Theoretically, it is the real instantaneous change occuring at time t, but for numerical-technical reasons (time step in calculations, discontinuous forcing), the ice sheet model cannot yield a meaningful instantaneous imbalance. Moreover, in reality, the relevance of the instantaneous imbalance is questionable because a strong interannual to decadal variability in the surface mass balance (precipitation minus ablation) generally overrides a more significant longer-term ice-sheet dynamic imbalance. We follow here the same approach as in *Huybrechts* [1994], and average the model outputs over the last 200 years to obtain the present evolution (Figure 3). This assumption is justified as the elastic response of the lithosphere involves microphysical processes that probably need several years or even decades to complete. Such a period can still be considered as instantaneous when compared to the characteristic times for the viscous response.

The total present-day change of Greenland ice volume is predicted to be close to zero. This confirms the basic result obtained in *Huybrechts* [1994] which was furthermore shown to be very robust to changes in ice-dynamic parameters. Despite this near overall equilibrium, the geographical pattern of ice thickness changes shows a clear distinction between a thickening of between 0 and +20 mm/ yr over the accumulation zone and thinning rates locally in excess of 100 mm/ yr at the southwestern and northeastern margins of the ice sheet (Figure 3, left panel). A comparison between these current ice thickness changes and the elastic bedrock response indicates a less regional response pattern than for the viscous part. This is because over the short periods characteristic of the elastic response, the asthenosphere (viscous response) does not have time to react and prevents the plate from bending. The elastic deformation therefore reduces to a more local compression or relaxation, proportional to the load variations as the lithosphere is assumed to be compressible. When, on the other hand, we consider the long-term viscous response (Figure 3, right panel), the outflow of mantle material allows the upperlying lithosphere to bend with a more pronounced regional character owing to the rigid stiffness of the plate. In terms of amplitude, the elastic instantaneous response remains small compared to the viscous response with a maximum elastic uplift of less than 0.5 mm/ yr compared to a maximum viscous response of about 6 mm/ yr. The main consequence of this difference is that the total present-day rate of uplift mainly depends upon past loading events rather than on the present changes of the ice sheet.

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