Using automatic differentiation to optimize parameters of shallow water models in tidal applications

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Outline

Goal: Unstructured tidal model with inverse estimation of parameters; adjoint model generation via automatic differentiation

Status of regional 2D tidal models

- Shallow water equations
- Triangular, unstructured meshes
- FV or FE with P_1^{NC} P_1 or P_1 P_1 discretization
- Different time-stepping schemes
- Inclusion of tidal potential
- Wetting & drying
- Clamped or Flather open boundary condition

Tuning of parameters:

- Bottom friction coefficient ($rH^{-1}|\mathbf{u}|\mathbf{u}$)
- Depth
- Open boundary values

Adjusting parameters manually is time consuming as the number of unknowns is large.

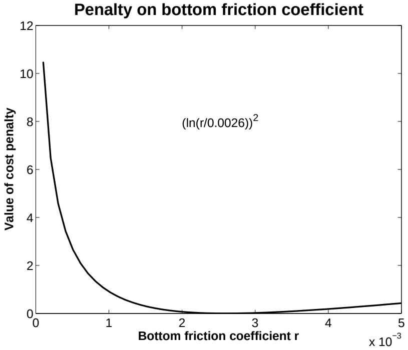
$\longrightarrow$ Inverse methods recontruct from the misfit between model results and observations the correct parameters.

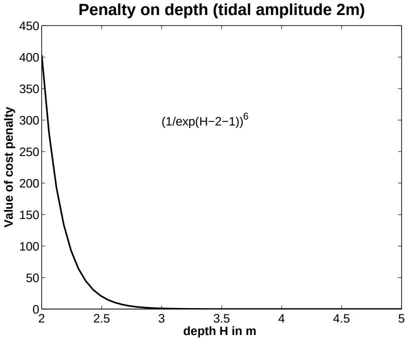
Adjoint model

We minimize a cost function, which is calculated after each tidal cycle:

$$J = \sum_{m=1}^{M} \left[\left(B_m^{obs} - B_m^{mod} \right)^2 + \left(D_m^{obs} - D_m^{mod} \right)^2 \right] s_p + \sum_{n=1}^{N} \left[\left(\ln \left(\frac{r_n}{2.6 \cdot 10^{-3}} \right) \right)^2 s_c + \left(\frac{1}{exp(H_n - A_n^{mod} - 1)} \right)^6 s_h \right]$$

B resp D are the real resp imaginary part of the oscillations. A is the amplitude. r is the bottom friction coefficient. H is the depth. s_p, s_c and s_h are scaling coefficients. M is the number of measurement points. N is the number of nodes.





... are a kind of regularization. It restricts the bottom friction parameter close to some initial guess and the bottom topography to stay in a range such that the depth is always positive.

Automatic differentiation ...

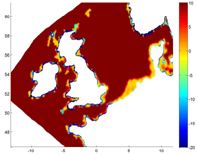
... is a technology for automatically adding statements for the computation of derivatives to computer programs.
www.autodiff.org
Advantages:

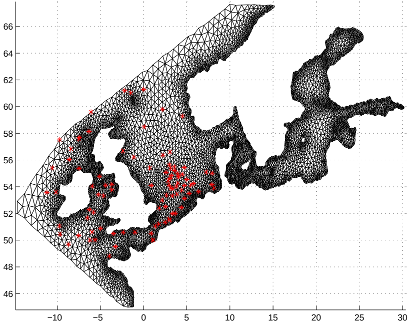
- Derivatives are accurate (contrary to FD methods)
- Adjoint model of the discretized equations
- Automatically generated adjoint models are easier to maintain
- Computation of Hessian for optimization algorithms is also possible
- Free software exists (TAMC, Tapenade, OpenAD,...)

Test setup

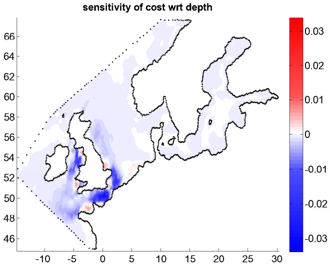
The adjoint model is generated of the explicit non-conforming FE code using TAMC.
(clamped boundary condition, no wetting and drying, minimal depth of 10m, no potential, only M_2 tidal forcing)

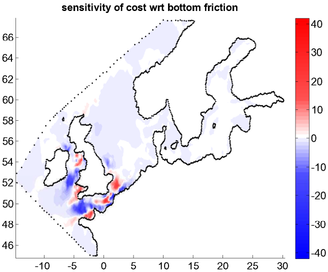
The scheme is tested on a very coarse mesh of the North- and Baltic Sea with only 7078 nodes. The cost function computes the misfit to 93 tidal gauges.

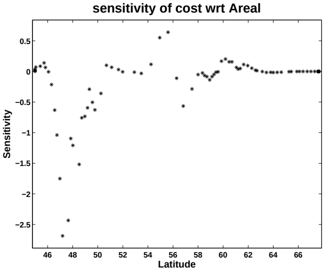


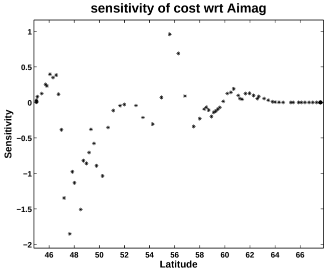


Initial gradient of the cost function

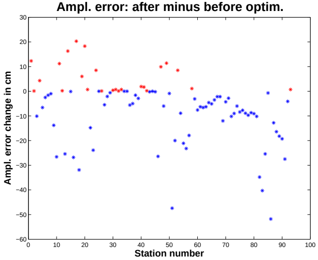


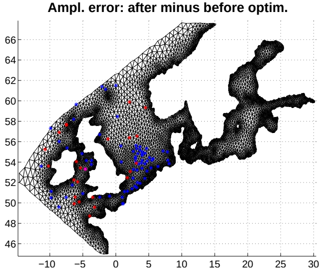


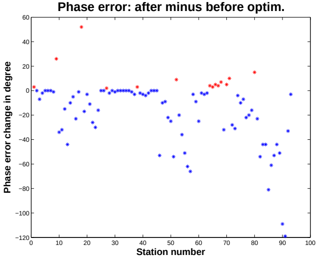


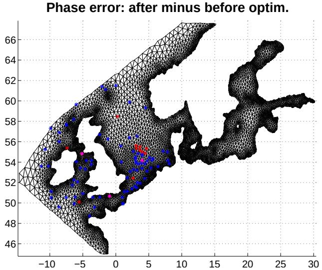


Reduction of error (optimized bottom topography and friction)

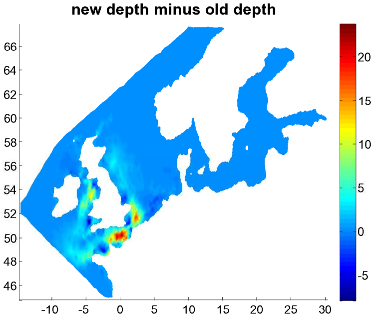


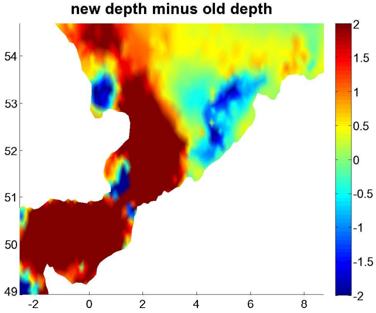






Optimized depth with respect to tide gauges



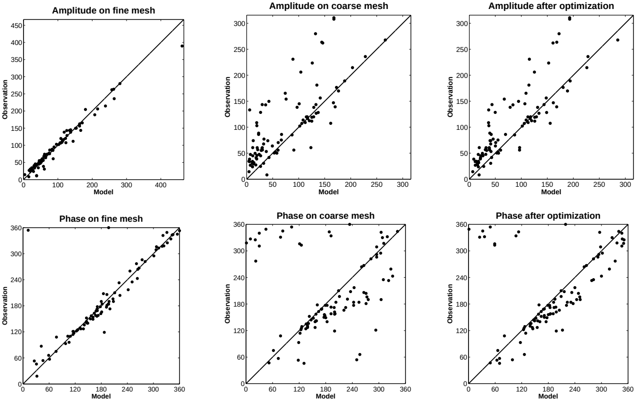


Conclusions

- Model is more sensitive to changes in open boundary values than to bottom friction and depth.
- Error reduction in more than two thirds of the stations.
- Optimized depth is consistent with our expectation.

Outlook

- Compare different AD tools to identify the most efficient
- Include wetting & drying
- Analyse the model dependencies to increase computational efficiency of the adjoint model
- Reduce initial error by taking better bathymetry and finer resolving mesh
- Optimize parameters for M_2 overtide simulation


Amplitude on fine mesh
Amplitude on coarse mesh
Amplitude after optimization
Phase on fine mesh
Phase on coarse mesh
Phase after optimization
Observation
Model