Ambiguity of signal–error estimates from normal equations

Grit Freiwald, Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany; Email: grit.freiwald@awi.de

1 Introduction

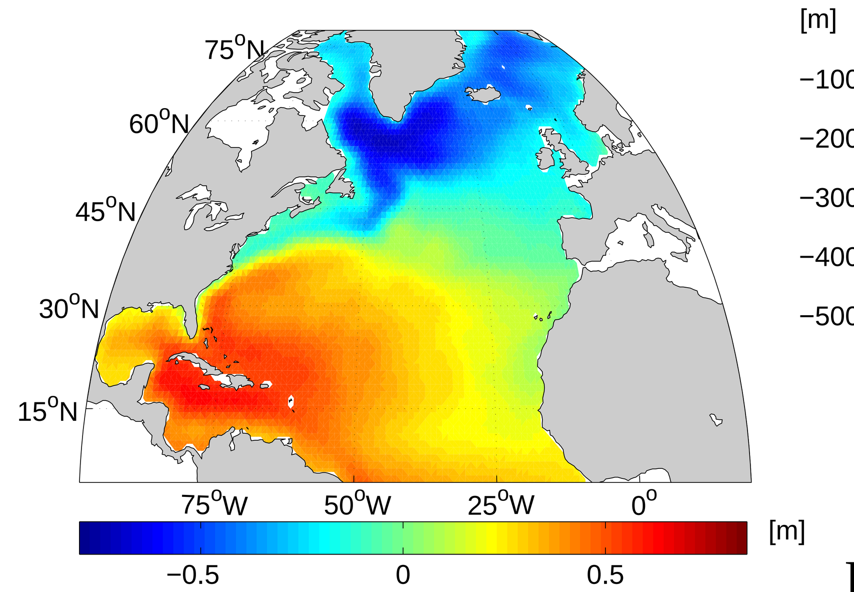
The inverse ocean model IFEOM assimilates Mean Dynamic Topography (MDT) data η_d from satellite observations.

Minimization of cost function: $J = \frac{1}{2}\sum_i J_i$, i=T,S,v,$\eta$,...

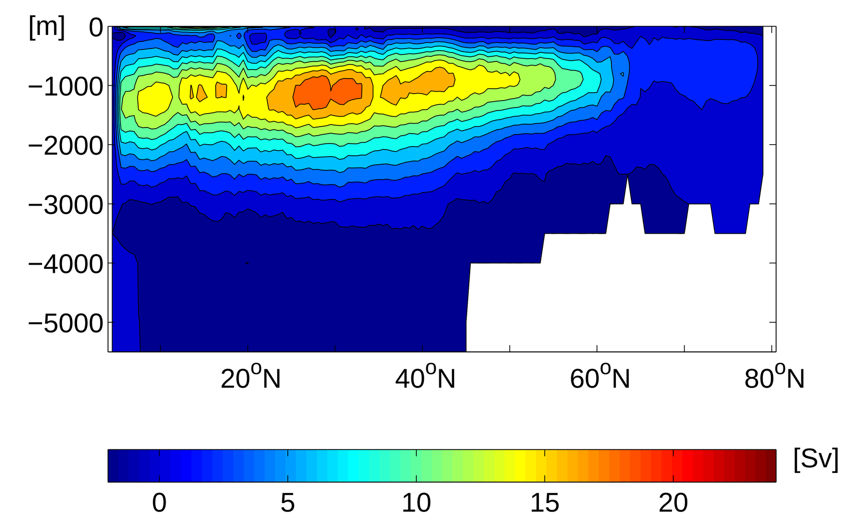
$$J_\eta = \frac{1}{\alpha}(\eta_\mathrm{m} - \eta_\mathrm{d})^T \mathbb{W}_\eta (\eta_\mathrm{m} - \eta_\mathrm{d}) \qquad (1)$$

For a discussion of the weighting factor α, please see other poster. $\mathbb{W}_\eta = \mathbb{C}_\eta^{-1}$ is the inverse MDT error covariance from the geodetic normal equations.

The matrix $\mathbb{W}_\eta$ is used to construct a filter $\mathbb{S}$ for the MDT data η_d. By inserting $\mathbb{I} = \mathbb{S}^{-1}\mathbb{S}$ into the geodetic observation equations, a new interpretation for MDT data and error covariance is derived.



First guess: MDT



First guess: Atlantic Meridional Overturning Circulation (AMOC)

The figures above show some oceanographic features of the first guess of the ocean model. The Gulf Stream and the AMOC are too weak (compare e.g. Johns et al., 1995, and Griffies et al.,2009).

2 General procedure

Observation equations: $A\eta_\mathrm{d} = \ell + v$, observation error covariance Σ

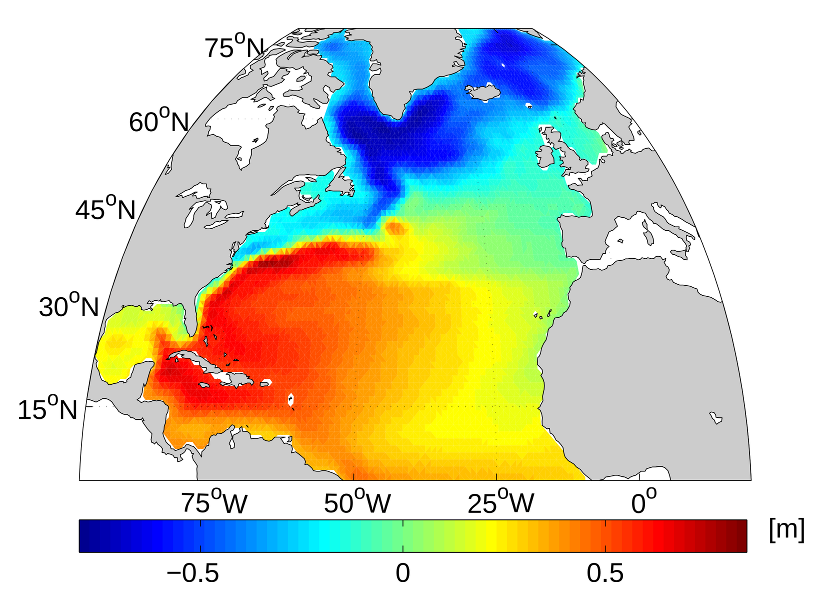
Generalized least squares:

$$A^T\Sigma^{-1}A\eta_\mathrm{d} = A^T\Sigma^{-1}\ell$$

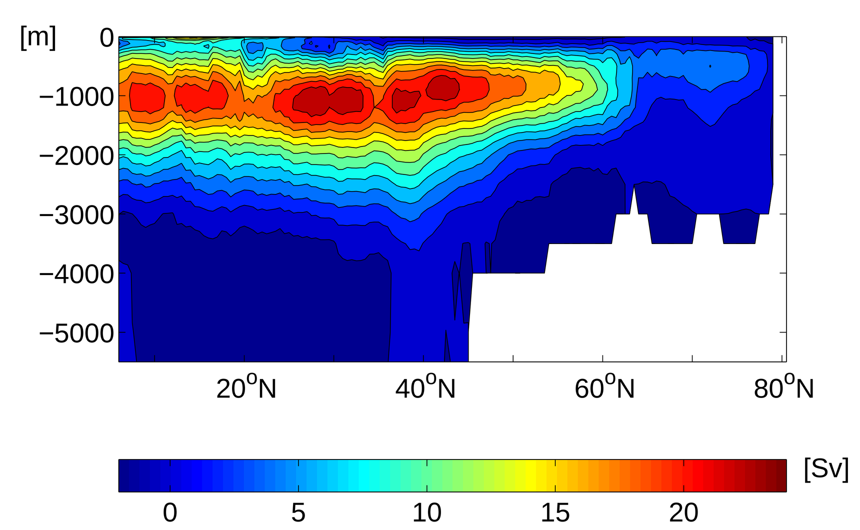
$$\underbrace{A^T\Sigma^{-1}A}_{\mathbb{W}_\eta}\eta_\mathrm{d} = \underbrace{A^T\Sigma^{-1}\ell}_{n}$$

$$\mathbb{W}_\eta\eta_\mathrm{d} = n$$

Use η_d as data and $\mathbb{W}_\eta$ as weighting matrix in the ocean model optimization (equation (1)) $\Longrightarrow$ Result I.



Result I: The MDT is improved by the standard procedure.



Result I: The AMOC is improved, however overestimated.

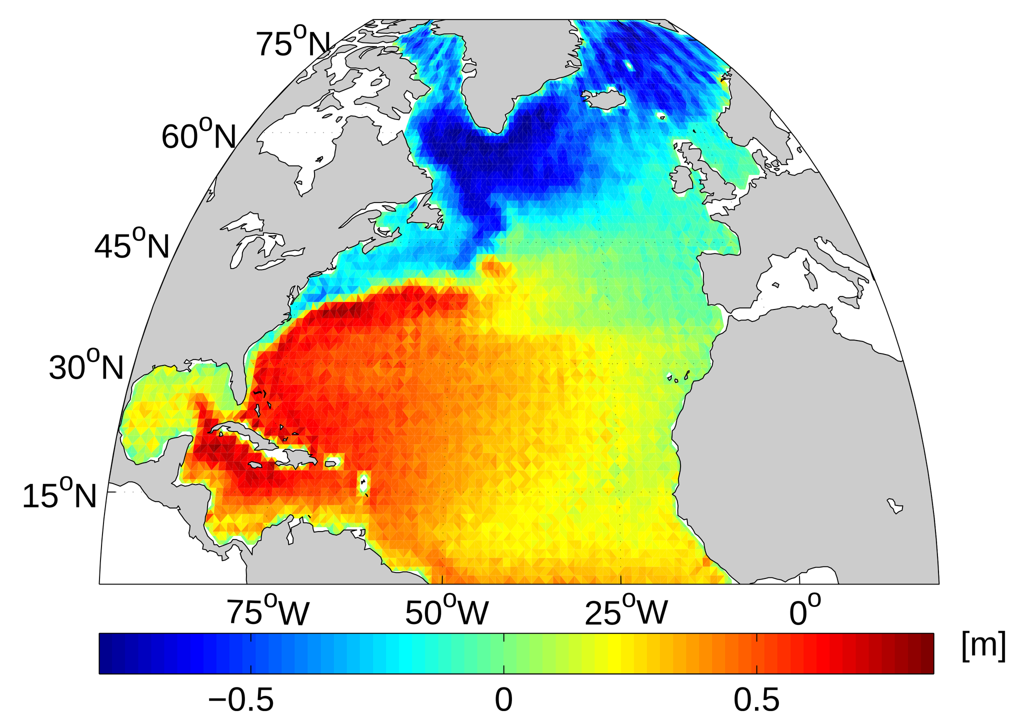
3 Smoothing filter $\mathbb{S}$

Inverse error covariance matrix $\mathbb{W}_\eta$ is symmetric and positiv definite

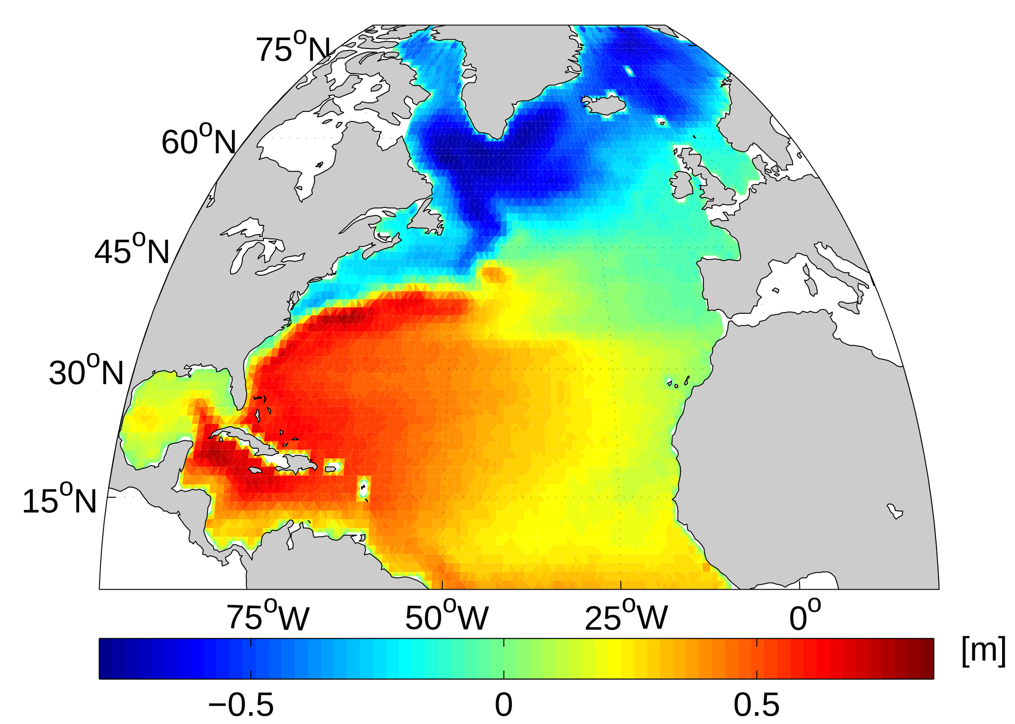
$\Longrightarrow$ A unique symmetric matrix square root exists: $\sqrt{\mathbb{W}_\eta} = \mathbb{V}$

Normalize rows of $\mathbb{V}$: $\mathbb{V} = \sqrt{\mathbb{D}}\,\mathbb{S}$ $\quad\Longrightarrow\quad$ $\mathbb{W}_\eta = \mathbb{S}^T\mathbb{D}\,\mathbb{S}$

The filter $\mathbb{S}$ is used to smooth the MDT data η_d.



Unsmoothed satellite MDT η_d



Smoothed satellite MDT $\mathbb{S}\eta_d$

Advantage: No arbitrary filter type, no arbitrary filter width!

4 Revised procedure

Using the smoothing filter $\mathbb{S}$, we derive a new interpretation of the geodetic observation equations:

Observation equations: $A\underbrace{\mathbb{S}^{-1}\mathbb{S}}_{=\mathbb{I}}\eta_\mathrm{d} = \ell + v$, error covariance Σ

Generalized least squares:

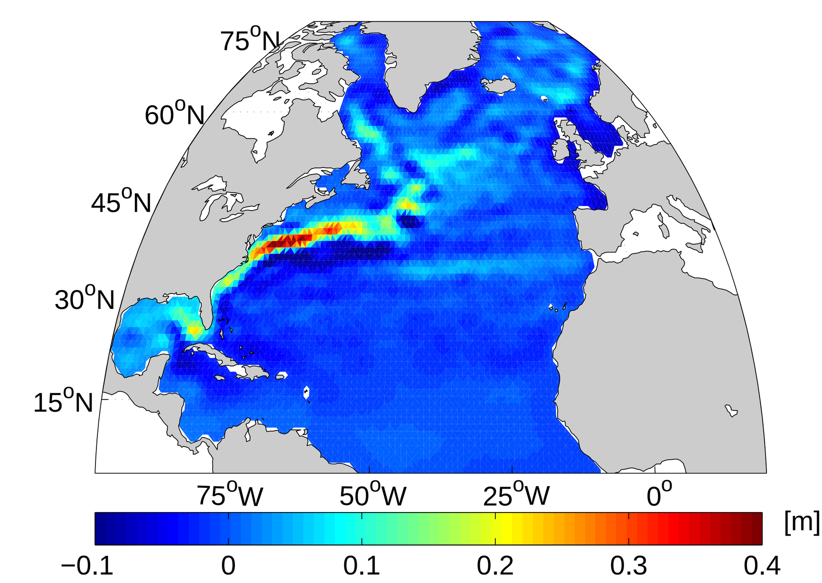
$$(A\mathbb{S}^{-1})^T\Sigma^{-1}A\mathbb{S}^{-1}\mathbb{S}\eta_\mathrm{d} = (A\mathbb{S}^{-1})^T\Sigma^{-1}\ell$$

$$\mathbb{S}^{-T}\underbrace{A^T\Sigma^{-1}A}_{\mathbb{W}_\eta}\mathbb{S}^{-1}\mathbb{S}\eta_\mathrm{d} = \mathbb{S}^{-T}\underbrace{A^T\Sigma^{-1}\ell}_{n}$$

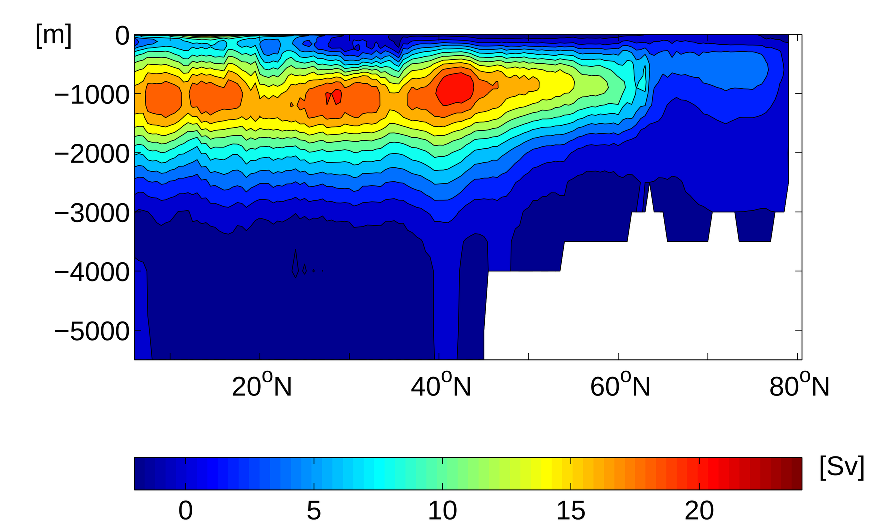
$$\underbrace{\mathbb{S}^{-T}\mathbb{W}_\eta\mathbb{S}^{-1}}_{\mathbb{D}}\mathbb{S}\eta_\mathrm{d} = \mathbb{S}^{-T}n \qquad \mathbb{W}_\eta = \mathbb{S}^T\mathbb{D}\,\mathbb{S}$$

$$\mathbb{D}\,\mathbb{S}\eta_\mathrm{d} = \mathbb{S}^{-T}n \quad\Longrightarrow\quad \hat{\boldsymbol{J}}_{\boldsymbol{\eta}} = (\boldsymbol{\eta}_\mathrm{m} - \mathbb{S}\boldsymbol{\eta}_\mathrm{d})^{\boldsymbol{T}}\mathbb{D}(\boldsymbol{\eta}_\mathrm{m} - \mathbb{S}\boldsymbol{\eta}_\mathrm{d})$$

Use $\mathbb{S}\eta_\mathrm{d}$ as data and $\mathbb{D}$ as weighting matrix in the ocean model optimization $\Longrightarrow$ Result II.



Δ MDT difference: Result II - Result I. The Gulf Stream is shifted northwards. No further large changes occur.



Result II: The AMOC is improved: It is not as intense as in the first approach and a distinct maximum at about 40-45°N is visible. ☺

5 Results

- A filter for the MDT was generated directly from the geodetic normal equations.
- No prior assumptions are made about the filter or the filter radius.
- The information content is shifted from the error covariance to the data themselves.
- The ocean model optimization with revised MDT data and error covariance improves the AMOC.