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## On the Impact of Nonlinearity on Ensemble Smoothing

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# Smoothers

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Filters (e.g. Ensemble Kalman filter)

- Estimate using observations until analysis time

Smoothers perform retrospective analysis

- Use future observations for estimation in the past

- Example applications:

- Reanalysis

- Parameter estimation

## Ensemble smoothing

- Smoothing is very simple (ensemble matrix  $\mathbf{X}_{k|k-1}^f$ )  
(see e.g. Evensen, 2003)

$$\text{Filter: } \mathbf{X}_{k|k}^a = \mathbf{X}_{k|k-1}^f \mathbf{G}_k$$

$$\text{Smother: } \mathbf{X}_{k-1|k}^a = \mathbf{X}_{k-1|k-1}^a \mathbf{G}_k$$

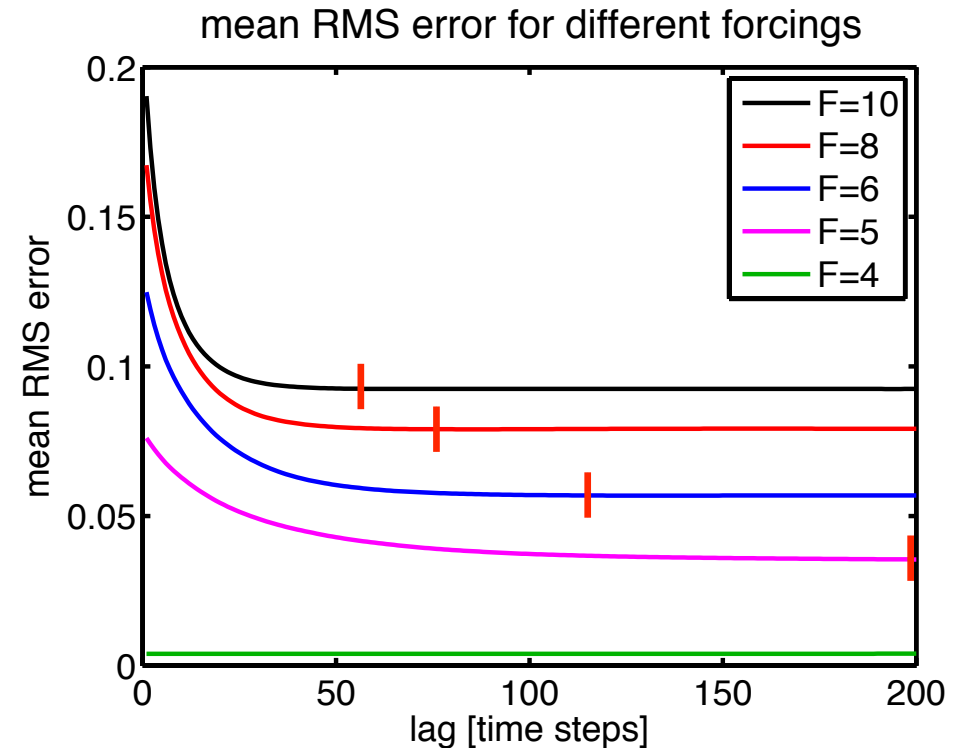
- Optimal for linear systems:
  - ➔ Forecast of smoothed state = analysis at later time
  - ➔ Each additional lag reduces error
- Not valid for nonlinear systems!
  - ➔ What is the effect of the nonlinearity?
  - ➔ Do ensembles just decorrelate? (see e.g. Cosme et al. 2010)

## Numerical study with Lorenz-96

- Cheap and small model (state dimension 40)
- Local and global filters possible
- Nonlinearity controlled by forcing parameter  $F$ 
  - Up to  $F=4$ : periodic waves; perturbations damped
  - $F>4$ : non-periodic
- Nonlinearity of assimilation also influenced by forecast length
- Experiments over 20,000 time steps
- Tune covariance inflation for minimal RMS errors
- Implemented in open source assimilation software PDAF  
(<http://pdaf.awi.de>)

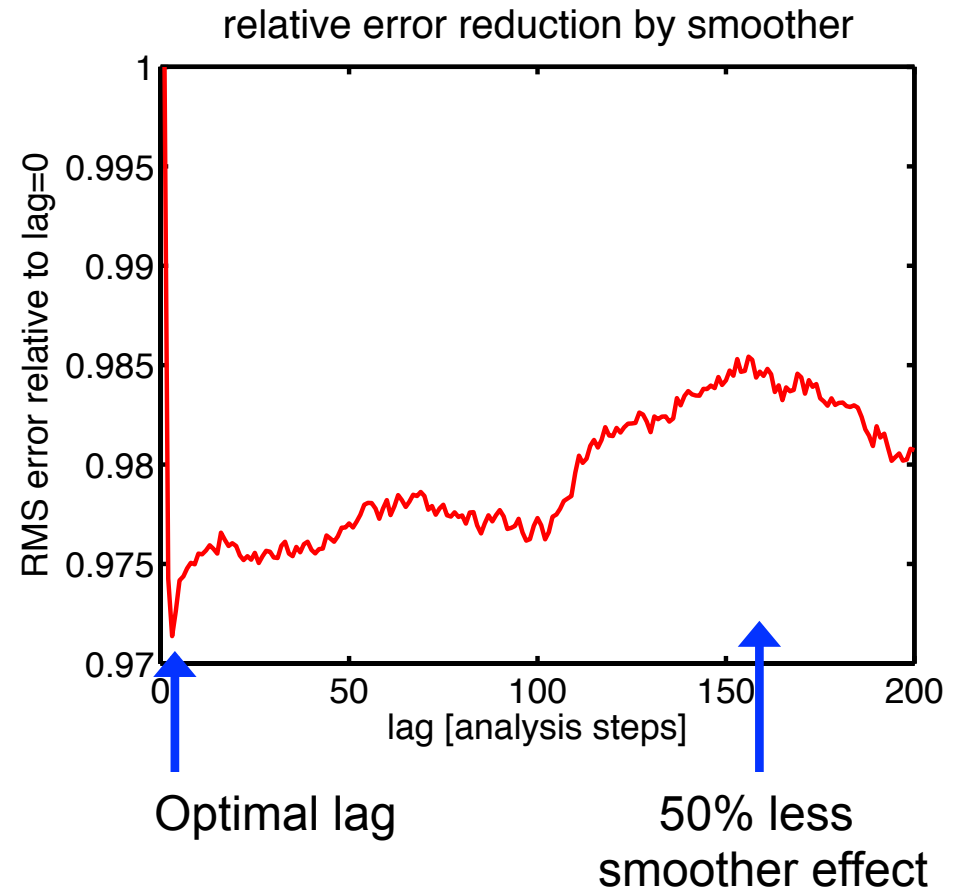
## Effect for forcing – optimal lag

- Assimilate at each time step
- Ensemble size  $N=34$
- Global ESTKF  
(Nerger et al., MWR 2012)
- Up to  $F=4$ 
  - very small RMS errors
- $F>4$ 
  - Strong growth in RMS
  - Clear impact of smoother
  - Optimal lag:  
minimal RMS error (red lines)

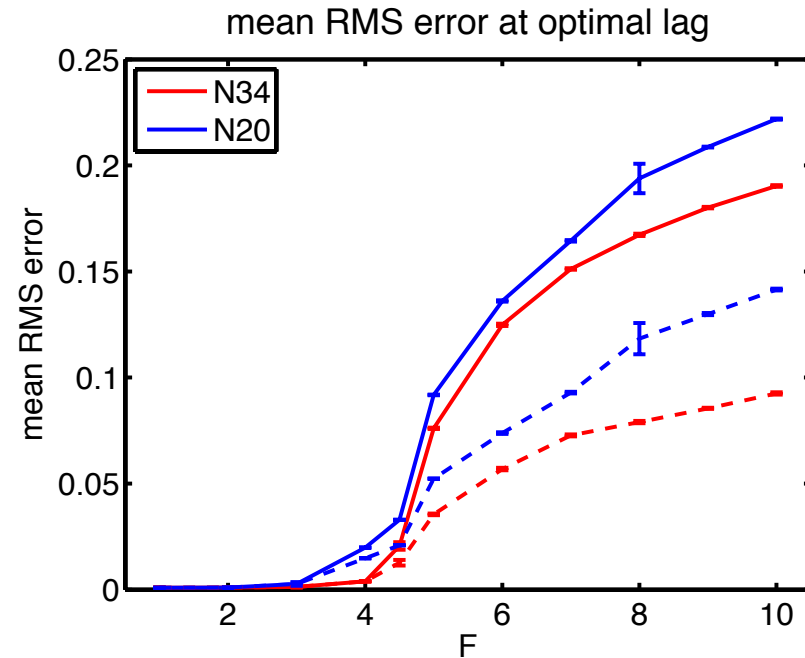
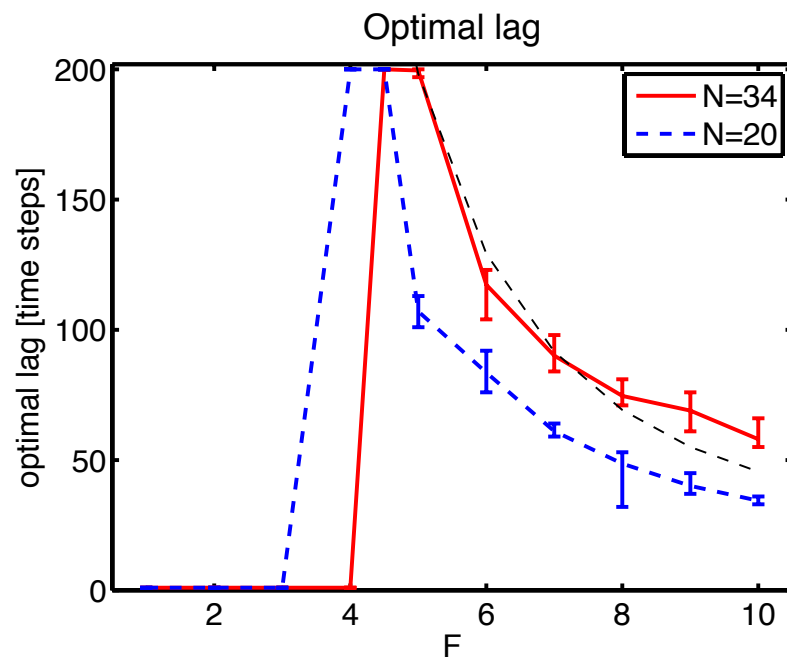


## Stronger nonlinearity

- $F=7$
- Forecast length: 9 steps
- Clear error-minimum at 2 analysis steps
  - ➔ the optimal lag
- Error increase beyond optimal lag (here 50%!)
  - ➔ spurious correlations

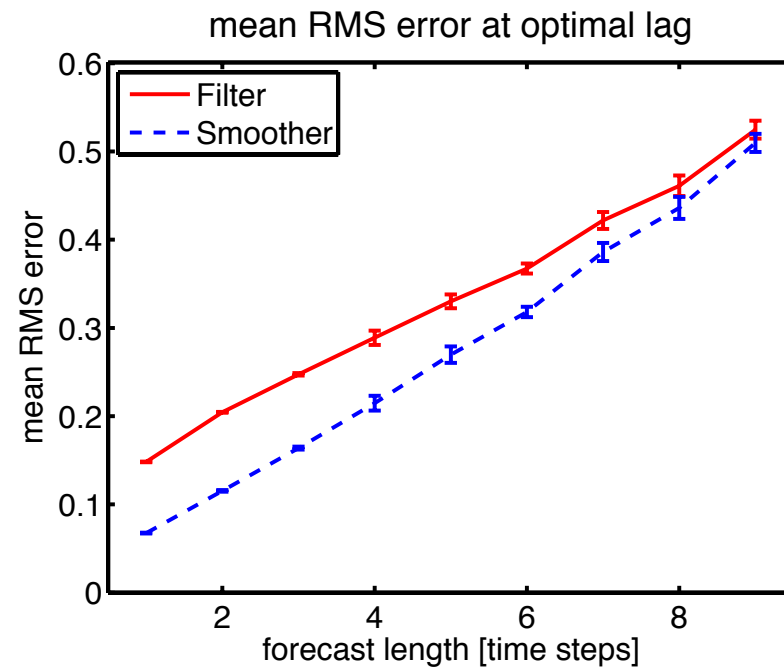
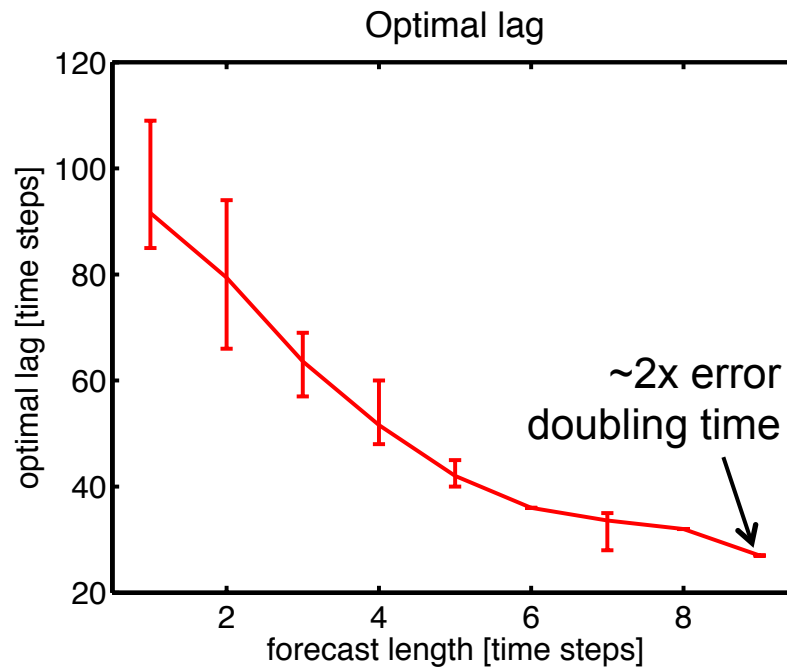


# Impact of smoothing



- Optimal lag (minimal RMS error)
  - Behavior similar to error-doubling time
- RMS error at optimal lag
  - Smoother reduces error by 50% for all  $F > 4$
- Effect of sampling errors visible with smaller ensemble

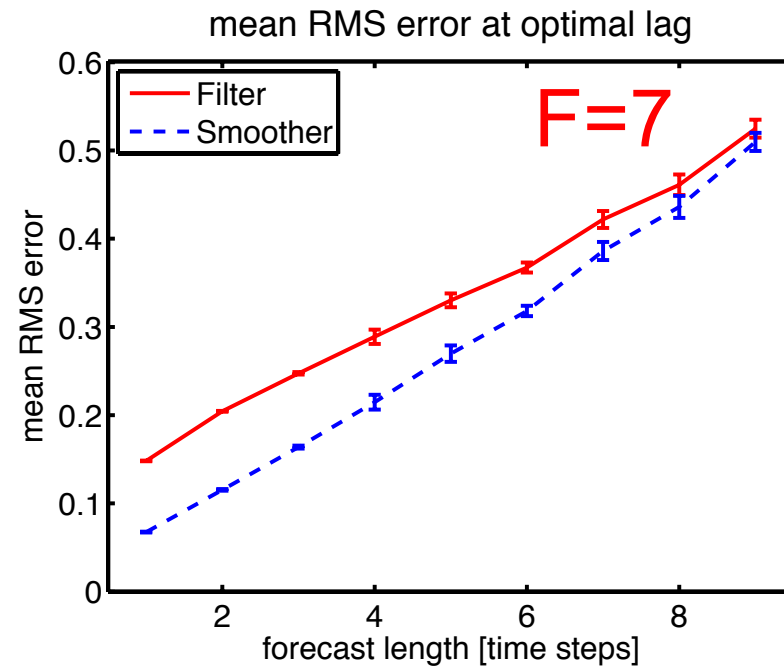
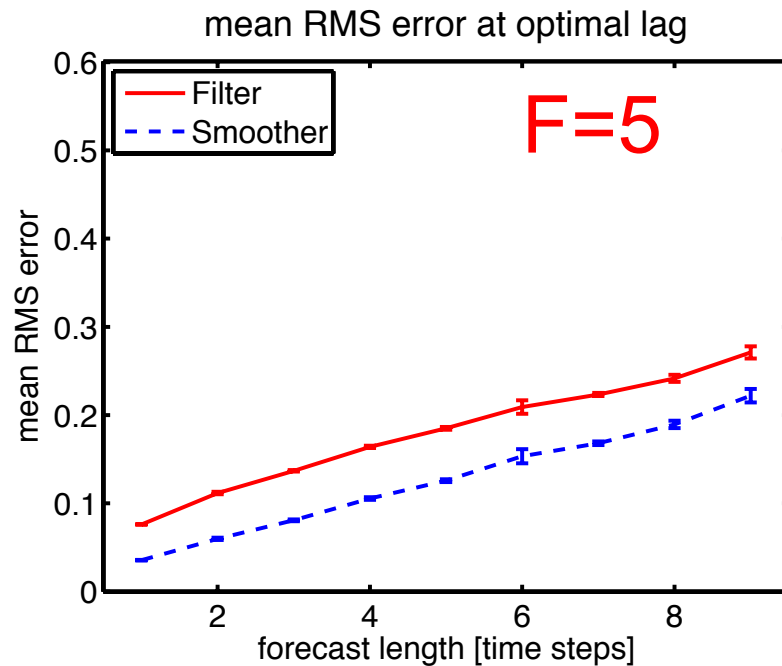
## Vary forecast length (F=7)



- Forecast length = time steps over which nonlinearity acts on ensemble
- Longer forecasts:
  - ➔ Optimal lag shrinks
  - ➔ RMS errors grow for filter and smoother
  - ➔ Improvement by smoother shrinks (depends on forcing strength)



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# Smoothing with global ocean model

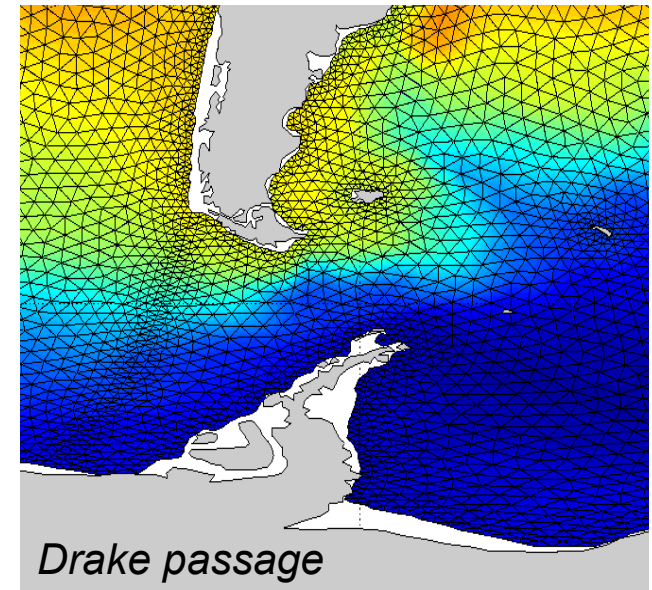
FESOM (Finite Element Sea-ice Ocean model, Danilov et al. 2004)

Global configuration

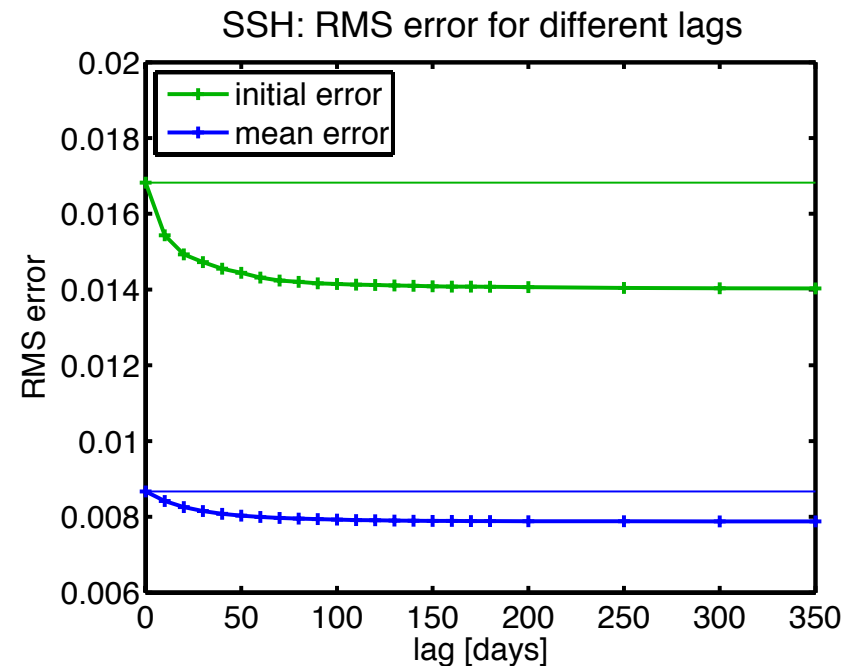
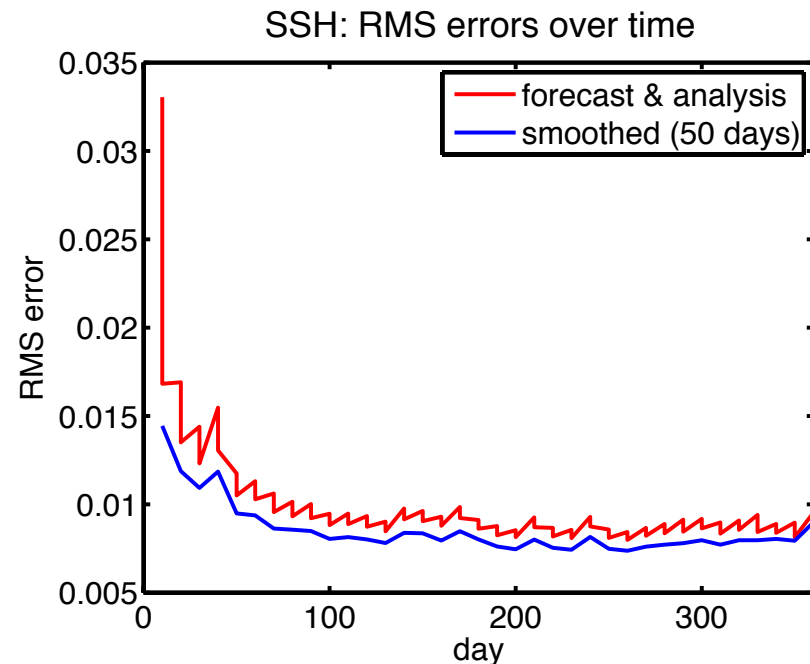
- 1.3° resolution, 40 levels
- Horizontal refinement at equator
- State vector size  $10^7$
- Weak nonlinearity

Twin experiments with sea surface height data

- Ensemble size 32
- Assimilate each 10th day over 1 year
- ESTKF with smoother extension and localization (Using PDAF environment as for Lorenz-96)
- Inflation tuned for optimal performance ( $\rho=0.9$ )



# Effect of smoothing on global model



## Typical behavior

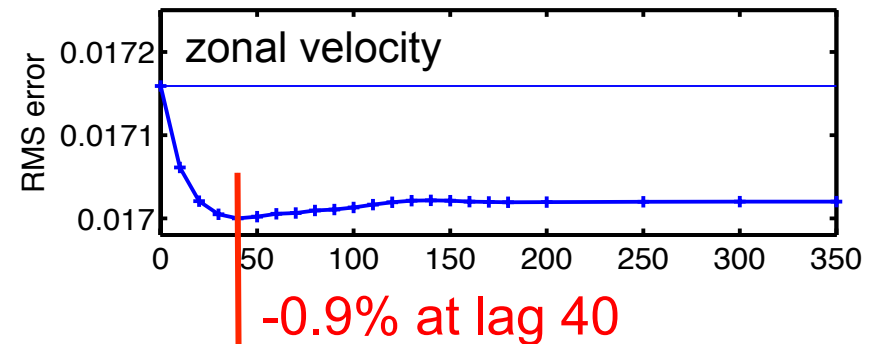
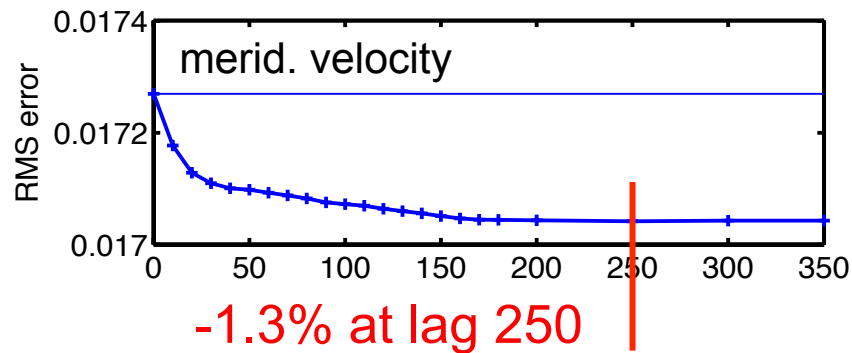
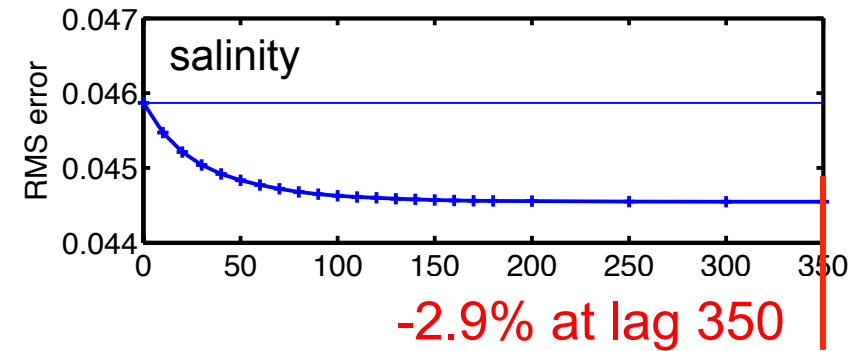
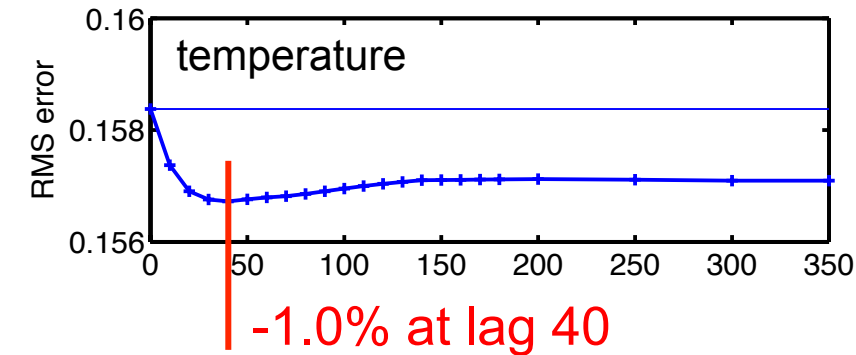
- RMSe reduced by smoother

## Error reductions:

- ~15% at initial time
- ~8% over the year

- Large impact of each lag up to 60 days
- Further reduction over full experiment (optimal lag = 350 days)

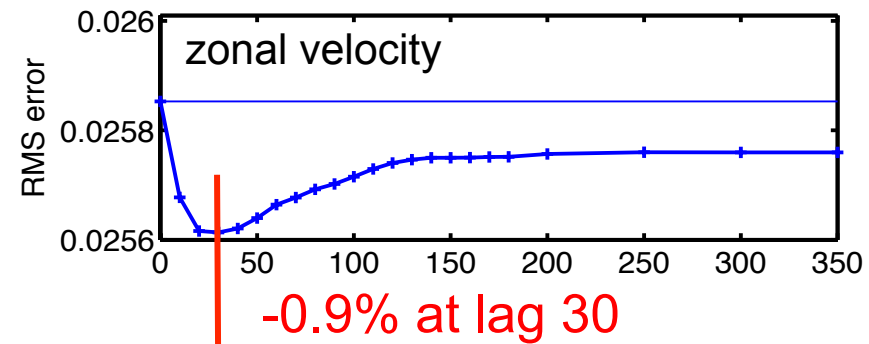
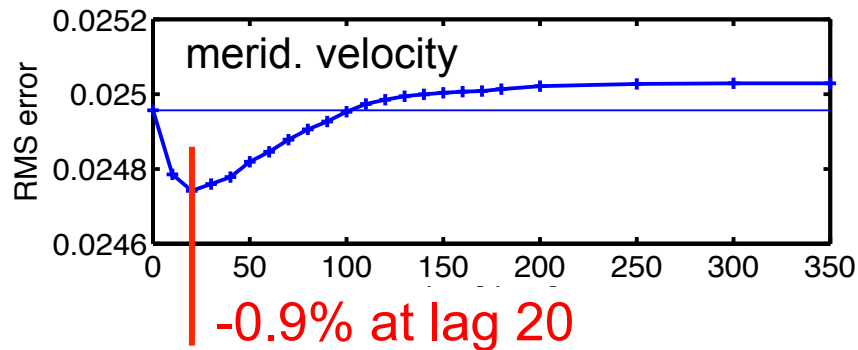
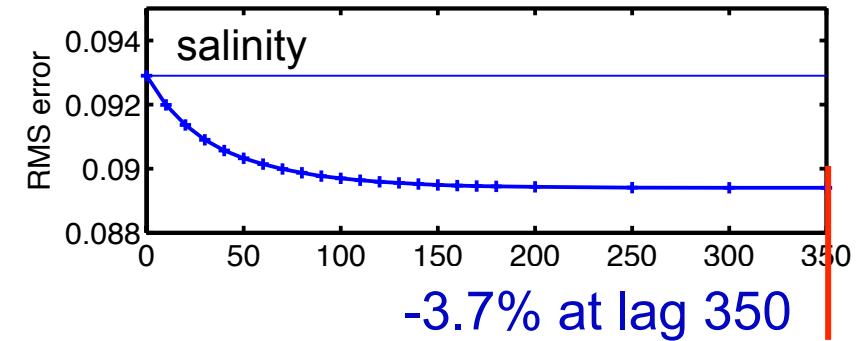
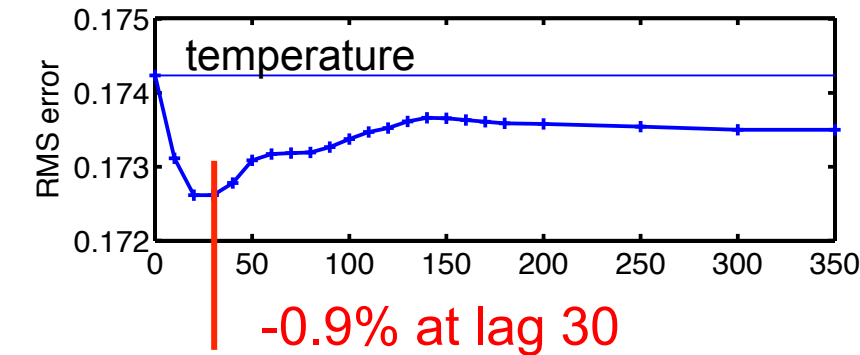
## Multivariate effect of smoothing – 3D fields



3D fields:

- Multivariate impact smaller & specific for each field
- Optimal lag specific for field
- Optimal lag smaller than for SSH

## Multivariate effect of smoothing – surface fields



Ocean surface:

- Relative smoother impact not larger than for full 3D
- Deterioration for meridional velocity at long lags
- ➔ What is the optimal lag for multivariate assimilation?

## Conclusion

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- Multivariate assimilation:
  - Lag specific for field
  - Choose overall optimal lag or separate lags
  - Best filter configuration also good for smoother
  
- Nonlinearity:
  - Introduces spurious correlations in smoother
  - Error increase beyond optimal lag
  - Optimal lag: few times error doubling time

**Thank you!**

## Web-Resources

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[www.data-assimilation.net](http://www.data-assimilation.net)



[pdaf.awi.de](http://pdaf.awi.de)