# Viscoelastic modeling of grounding line migration

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## Introduction

Grounding line dynamics is an important factor for modeling ice sheet discharge.
Tides play an important role as they modulate the flow of ice streams even far upstream. Grounding lines migrate in response to tidal forcing (as shown in Fig. 1 and Fig. 2), but the exact mechanisms and consequences are not yet understood in detail.
We investigate the influence of tides on the dynamics of ice sheet – ice shelf systems and grounding line migration using a continuum mechanical approach by choosing a viscoelastic material model motivated by short and long term response of ice to load. For this purpose a fully coupled viscoelastic full-Stokes ice flow model was implemented in the finite element software COMSOL Multiphysics.

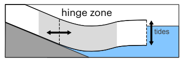



**Fig. 1**
**Grounding line – ocean interaction**

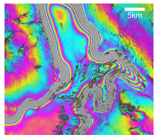



**Fig. 2**
**Quadruple InSAR image of the Wilkins Ice Shelf; the fringe belt represents the hinge zone (Moll 2007)**

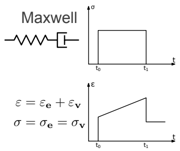
## Theory

The field equations for the 2D full Stokes are

$$\operatorname{div} \mathbf{v} = 0 \qquad \text{mass balance}$$

$$\operatorname{div} \sigma + \mathbf{f} = 0 \qquad \text{momentum balance}$$

On short timescales, as present in tidal forcing, we need to account for the elastic character of glacier ice. A Maxwell rheology model represents the elastic response on short timescales while on long timescales the viscous contribution dominates.




Maxwell:

$$\dot{\varepsilon} = \frac{1}{E}\dot{\sigma} + \frac{1}{\eta}\sigma$$

Glen:

$$\eta(\dot{\varepsilon}_e) = \frac{1}{2}(EA)^{-1/n}\dot{\varepsilon}_e^{(1-n)/n}$$

**Fig. 3**
**Ice rheology: Maxwell response to sudden load**

## Numerical model

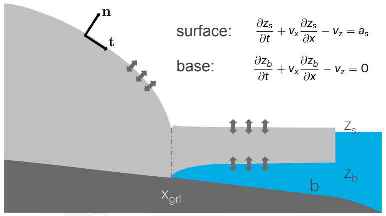



**Fig. 4**
**Model geometry and boundary conditions**

In contrast to similar models (e.g. Gudmundsson 2011) we directly solve for the position of the ice–atmosphere and ice–water interfaces using the kinematic boundary conditions. This allows us to prescribe the dynamic boundary conditions in the most direct way:

surface

$$\mathbf{n}\cdot(\sigma\cdot\mathbf{n})|_{z_s} = 0$$

$$\mathbf{t}\cdot(\sigma\cdot\mathbf{n})|_{z_s} = 0$$

calving front

$$\mathbf{n}\cdot(\sigma\cdot\mathbf{n})|_{cf} = p_w(z,t)$$

$$\begin{cases} p_w(z,t) = \rho_{sw}\mathbf{g}(z_sl(t)) - z), & z < z_{sl}(t) \\ p_w(z,t) = 0, & z \geq z_{sl}(t) \end{cases}$$

floating base

$$\mathbf{n}\cdot(\sigma\cdot\mathbf{n})|_{z_b} = -\rho_{sw}\mathbf{g}(z_{sl}(t) - z_b)$$

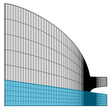
$$\mathbf{t}\cdot(\sigma\cdot\mathbf{n})|_{z_b} = 0$$

grounded base

$$\mathbf{t}\cdot(\sigma\cdot\mathbf{n})|_{z_b} = C|v_b|^{m-1}v_b$$

$$\mathbf{v}\cdot\mathbf{n}|_{z_b} = 0$$

**Fig. 5**
**Illustration of the contact problem and computational mesh**



We consider the decision if the ice is grounded or floating as a contact problem. Hence the position of the grounding line is not known in advance, but part of the numerical solution.

floating $\quad z_b(x,t) > b(x)$

grounded $\quad z_b(x,t) = b(x)$ and $-\sigma_{nn}|_b > p_w(z_b,t)$

We begin with evolving a steady state geometry and run subsequently experiments with tidal forcing enabled. We set up two geometries with a steep and flat bed as shown below.

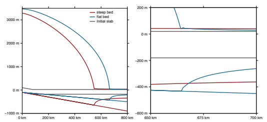


**Fig. 5**
**Initial steady state geometry for a steep and a flat bed**

## Results

In the continuation we show results with non-linear sliding (m=1/3). In the first setup the grounding line does not migrate. When forced with the S2 (12 h) and M2 (12.42 h) tidal constituents (peak to peak amplitude of 1.2m), we observe – as expected for viscoelastic materials – a phase shift, as well as a non-linear interaction (Fig. 7), which leads to a perturbation of the horizontal flow velocity close to the Msf (14.76 d) constituent.

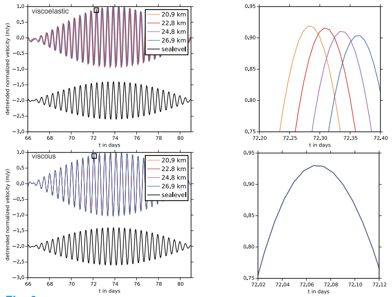



**Fig. 6**
**Detrended normalized horizontal surface velocities at different distances upstream the grounding line (left); with zoom into a subset (right); top: Maxwell rheology; bottom: purely viscous control run**

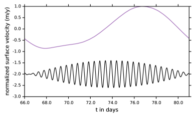
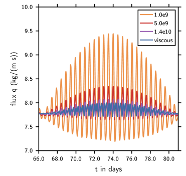


**Fig. 7**
**Surface velocity at a distance of 87km — velocity perturbation close to Msf frequency.**

When looking at the mean flux over the grounding line, we observe an asymmetry leading to an increase of total flux (Fig. 8).
Hence not including tides and viscoelasticity into ice models we neglect significant contribution for the estimation of the flux across the grounding line and the resulting mass balance. For our experimental setup this difference depends on the elastic parameter and we obtain a maximal difference of 3.75%.

**Fig. 8**
**Flux across grounding line for different elastic parameters**




The second setup leads to grounding line migration as the peak to peak amplitude is 20m. Two processes control ice flow velocity variations: Uplifting of the ice shelf leads to retreat of the grounding line and therefore less area of the ice base is in contact with the bedrock. This leads to smaller basal shear stress, resulting in an increase in flow velocity. Additionally, high tide causes increased normal stress at the ice – water boundary, which reduces the ice flow.

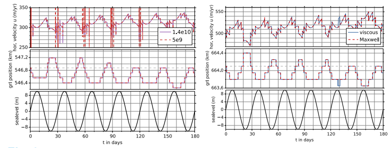


**Fig. 9**
**left: grounding line migration on steep bed. Two different elastic parameters**
**right: grounding line migration on flat bed.**

When comparing migration on different slopes, we observe a larger migration on the steeper bed, which is in contrast to the usual assumption. This can be explained by the different steady state geometry and therefore distinct flow field.

We also observe a general retreat of the grounding line due to tidal forcing. This implies that tides possibly lead to a different equilibrium of the grounding line position.

## Conclusions

- Using a fully coupled viscoelastic (Maxwell, full-Stokes) model and a steady state start geometry, leads to similar results as those of Gudmundsson (2011)
- Verification of the model shown by phase shift in viscoelastic and purely viscous runs
- Tides lead to increased net flux across the grounding line of ca. 1-3%
- Smaller slope does not simply lead to larger migration
- Tides and viscoelastic constitutive relations lead possibly to a new equilibrium state of the grounding line position

**References**
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