Adjoint Sensitivity of an Ocean General Circulation Model to Bottom Topography

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1 Overview

Numerical solutions of ocean general circulation models are determined by many different parameters. So far, state estimation based on inverse methods, such as that of the ECCO consortium, use surface boundary conditions and, for integrations shorter than the time for baroclinic adjustement processes to complete, initial conditions as canonical control variables. Other parameters, for example diffusivities, lateral boundary conditions (free-slip, no-slip, etc.), and bottom topography, are formally assumed to be known. However, it is not clear what their "correct" values in coarse resolution models are. Bottom topography, for example is not known accurately in large regions of the ocean. Even where it is known, its representation on a coarse grid is ambiguous and may add numerical artifacts to the ocean model's solution. Here, we extend the approach of Losch and Wunsch (2003), who use topography as a control variable in a simpler model, to a full general circulation model Losch and Heimbach (2006).

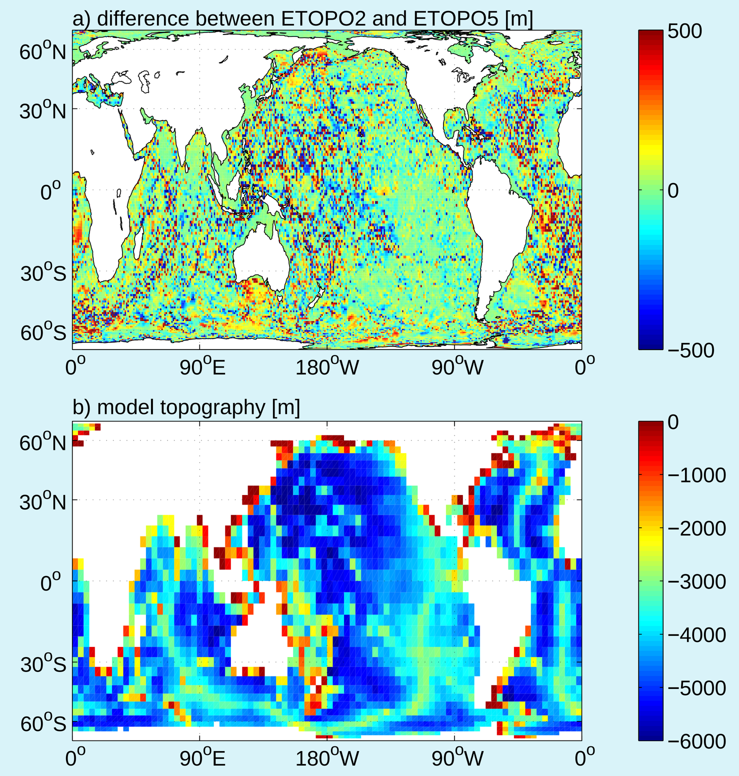

Figure 1: a) Difference between two sea floor topographies (ETOPO2 minus ETOPO5), smoothed with a 1° radial block average. Not only are the differences locally larger than 1000 m, but also is ETOPO2 systematically shallower in parts of the Southern Ocean, e.g., south of the Australian continent. b) Topography of numerical model in this study.

2 MITgcm and its adjoint

The M.I.T. General Circulation Model (MITgcm) is rooted in a general purpose grid-point algorithm that solves the Boussinesq form of the Navier-Stokes equations for an incompressible fluid, hydrostatic or fully non-hydrostatic, in a curvilinear framework (in the present context on a three-dimensional longitude (λ), latitude (φ), depth (H) grid. The algorithm is described in Marshall et al. (1997) (for online documentation and access to the model code, see MITgcm Group, 2002)).

The MITgcm has been adapted for use with the Tangent linear and Adjoint Model Compiler (TAMC), and its successor TAF (Transformation of Algorithms in Fortran, Giering and Kaminski, 1998). Efficient (w.r.t. CPU/memory), exact (w.r.t. the model's transient state) derivative code can be generated for up-to-date versions of the MITgcm and its newly developed packages in a wide range of configurations (Heimbach et al., 2002, 2005). In addition to the general hurdles that need to be tackled for efficient exact adjoint code generation, inclusion of bottom topography as a control variable added further complexities to the problem:

1. expressions involving the product of an element of the model state and topographic masks are now quadratic w.r.t. algorithmic differentiation;
2. the elliptic operator to solve for the surface pressure/height field now looses its self-adjoint property, and needs to be explicitly adjointed as well.

3 Drake Passage Transport

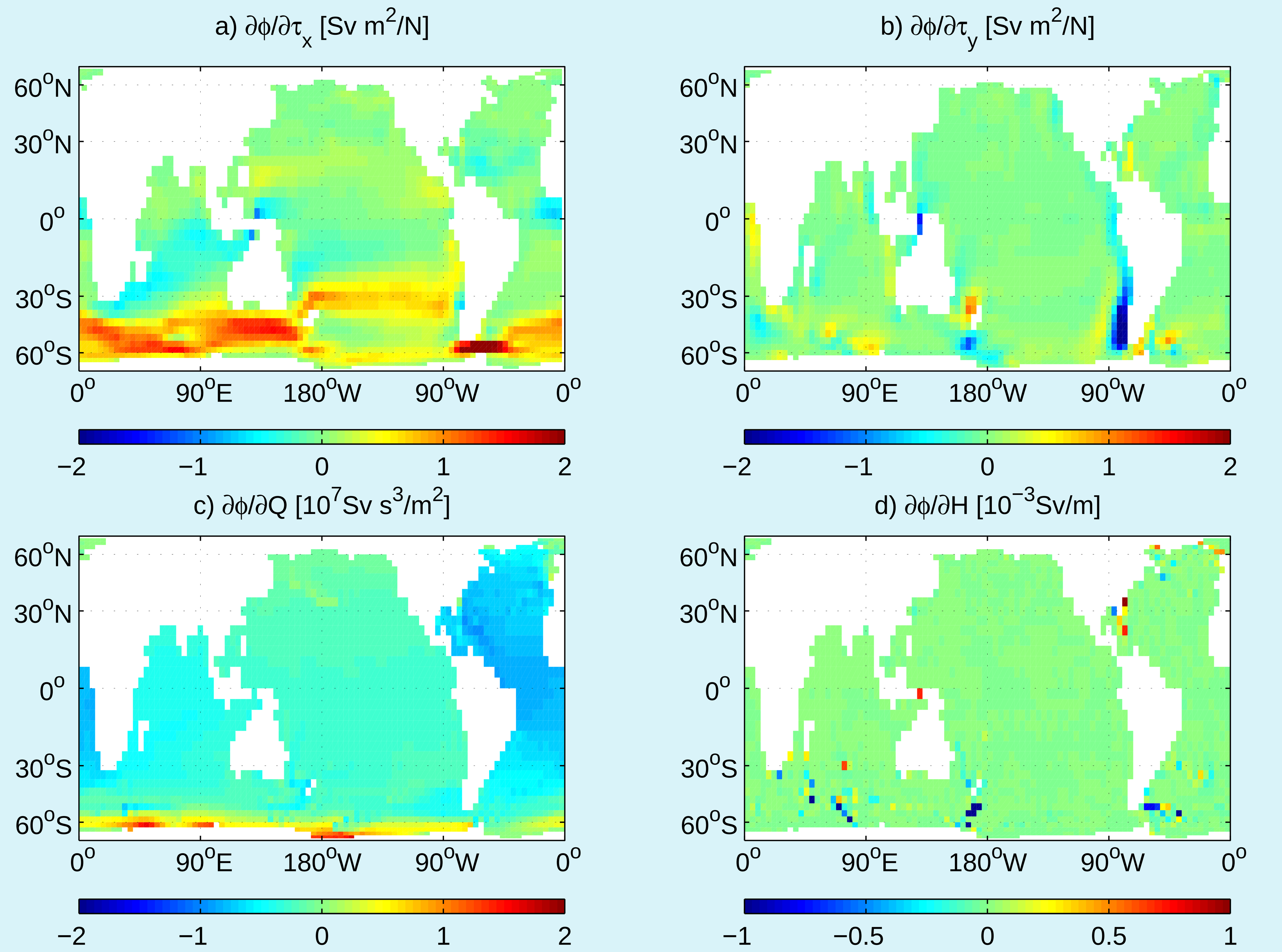

Figure 2: The Drake Passage is a sensitive "choke point" in the Southern Ocean. We show the adjoint sensitivity of volume transport through the Drake Passage ϕ to surface stresses ($\partial\phi/\partial\tau_x$, $\partial\phi/\partial\tau_x$), surface buoyancy fluxes ($\partial\phi/\partial Q$, $\partial\phi/\partial EmP$), and bottom topography ($\partial\phi/\partial H$). The volume transport is "measured" at the end of a 100 year spin-up integration starting from rest.

4 Atlantic Overturning Streamfunction at 24° N

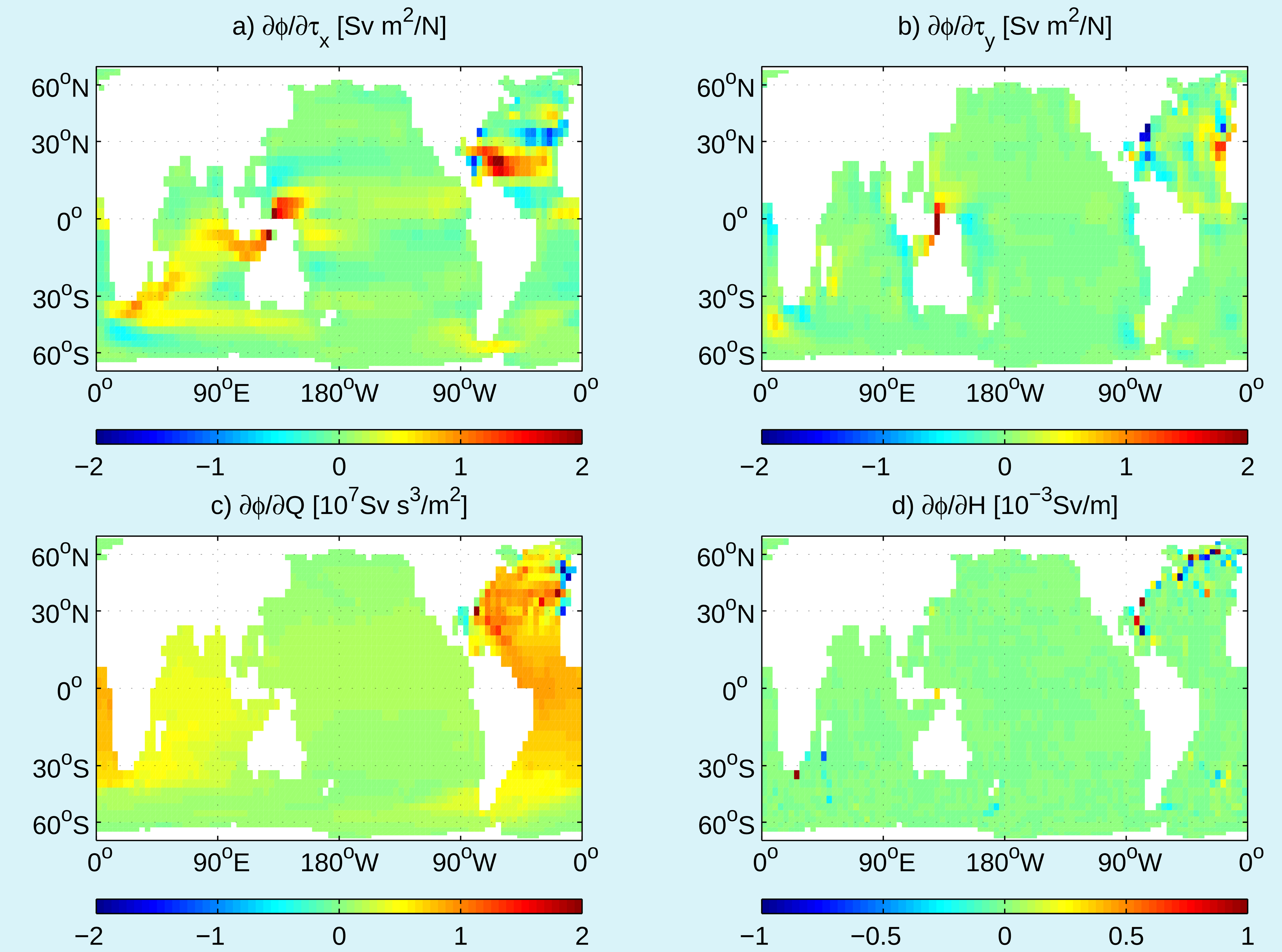

Figure 3: Adjoint sensitivity of meridional overturning strength ϕ at 24°N to surface stresses, surface buoyancy fluxes and bottom topography. The overturning strength is "measured" at the end of a 100 year spin-up integration starting from rest. Other than for the Drake Passage transport, the largest sensitivities are found close to the location of the objective function.

5 Sensitivity to temporal scale and spatial resolution

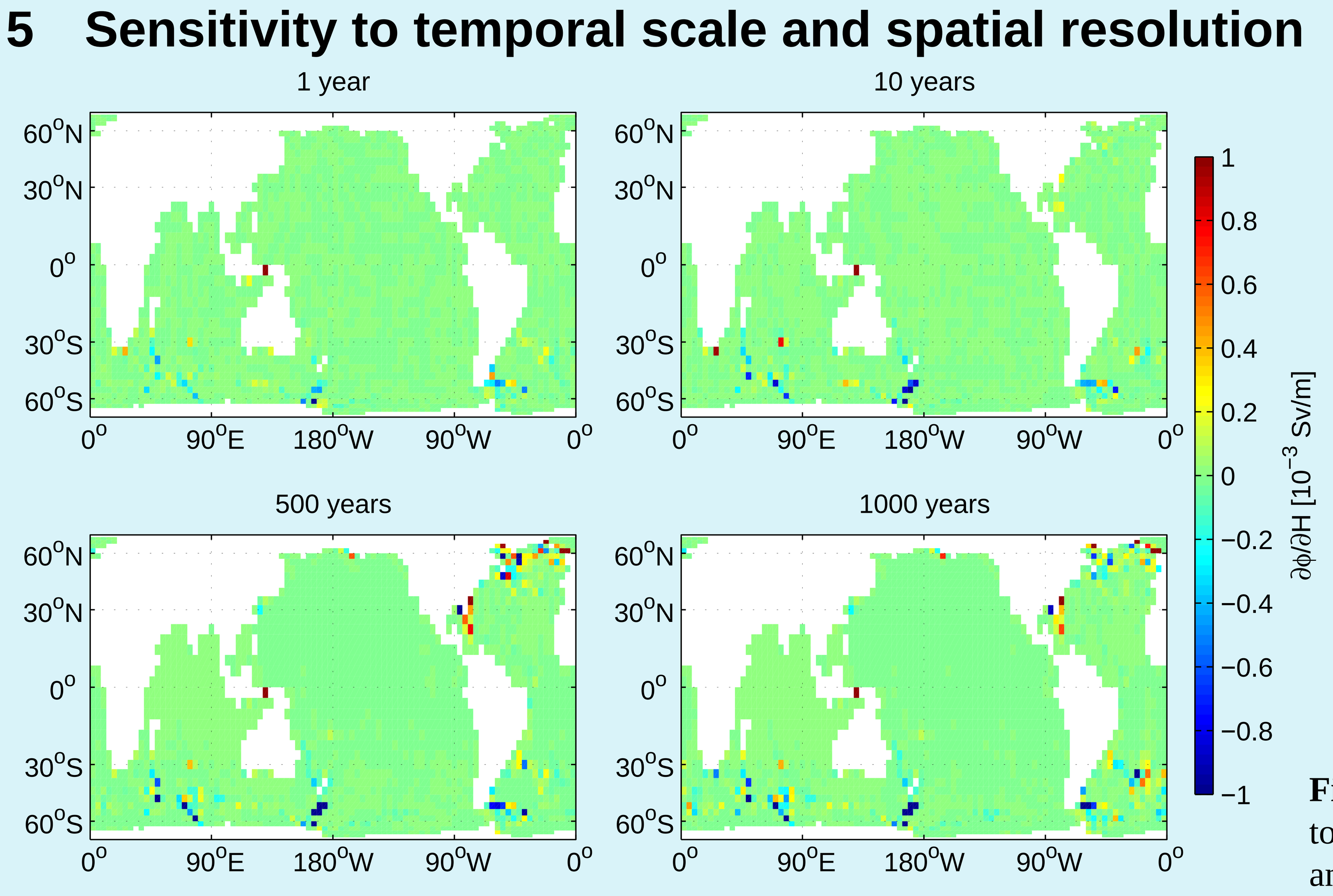

Figure 4: The adjoint sensitivity to bottom topography in Fig. 2 depends only marginally on the length of the spin-up integration. The main patterns of sensitivity near the Drake Passage, the Kerguelen Plateau, the Macquarie Ridge and the Indonesian Throughflow are robust with respect to length of the spin-up integration; shown are runs of 1, 10, 500, and 1000 years. It is worth noting that with increasing integration length the Drake Passage transport becomes sensitive to the topography of the North Atlantic (see also Fig. 2).

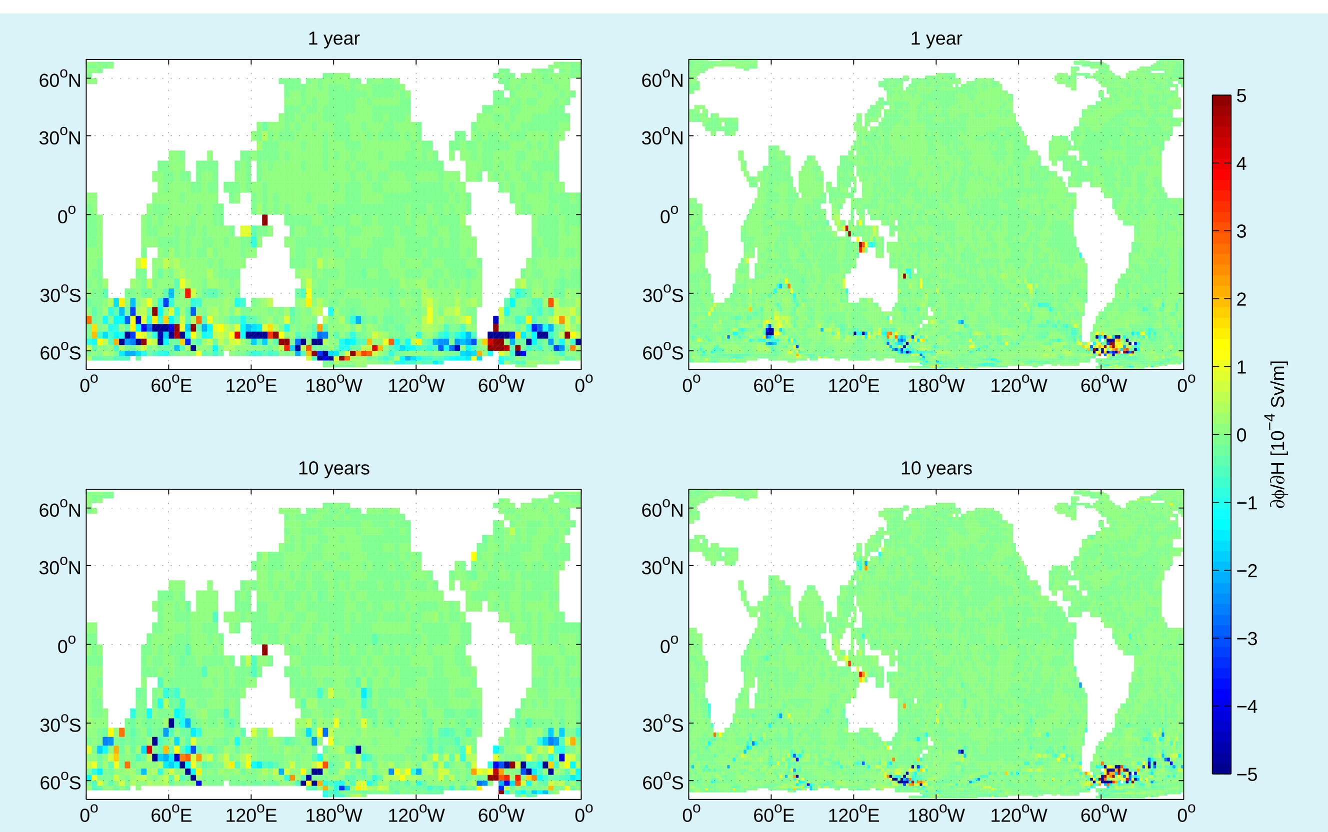

Figure 5: The adjoint sensitivity of volume transport through the Drake Passage to bottom topography is indeed sensitive to specifics of the model configuration such as horizontal and vertical resolution. For a configuration with twice the resolution (2 by 2 degrees) than in the previous experiments, the spatial pattern of the adjoint sensitivity is far more detailed than in the coarse resolution experiment (Fig.4), but the large scale patterns remain similar. The Antarctic Circumpolar Current is steered around the Kerguelen Plateau in this experiment, thus reducing the sensitivity seen in the coarse resolution experiment. The volume transport is "measured" at the end of a 10 year spin-up integration starting from rest.

6 Discussion

One can assess the relative importance of different control variables by multiplying the respective gradient by their a priori uncertainty estimates, i.e.

$$\Delta\phi = \frac{\partial\phi}{\partial u}\Delta u$$

with u a control variable element and Δu some a priori uncertainty estimate. The following table illustrates that in places, the effect of bottom topography sensitivities are of similar magnitude as that of wind stress sensitivities (c.f. Fig. 2).

control variable u	τ_x	H
Δu	0.1 $\mathrm{N/m^2}$	100 m
$\partial\phi/\partial u$	2 $\mathrm{Sv\,m^2/N}$	$-2\cdot10^{-3}\,\mathrm{Sv/m}$
$\Delta\phi$	0.2 Sv	-0.2 Sv
location	Drake Passage	Kerguelen Plateau

This information can be used to systematically adjust the geometry of a coarse (w.r.t. to the actual topography) general circulation model. Eventually, bottom topography and further "unorthodox" parameters such a lateral boundary conditions (no slip, free slip) will be included in the control vector of a state estimation problem.

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