

# Intercomparison between several finite element and finite volume approaches to model the North Sea tides

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Overall goals:

- ▶ unstructured, non-linear shallow water models
- ▶ adjoint model generation
- ▶ optimization of model parameters

What is the focus of this presentation?

- ▶ influence of spatial discretization (FE, FV)
- ▶ computational efficiency

## Unstructured grid models

finite volume (FV): easy to implement, less accurate in space

finite element (FE): implementation more elaborate

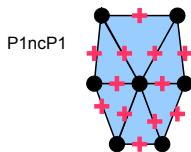
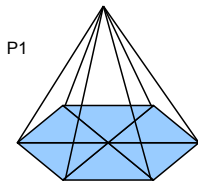
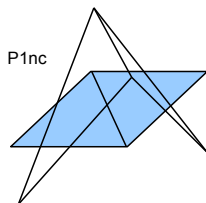
FV:

- ▶ Chen et al (FVCOM)
- ▶ Casulli&Walters (UnTRIM)

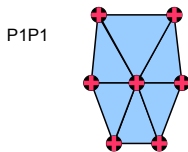
FE:

- ▶ wave continuity equation models (ADCIRC, QUODDY, MOG2D, T-UGO)
- ▶ other models (TELEMAC-2D, P1P1, NC)

## finite elements



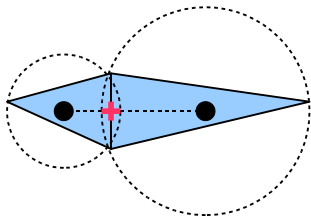
+ velocity      ● elevation



- ▶  $P_1^{nc}P_1$  (NC): approx. 3x more edges than nodes
- ▶  $P_1P_1$ : pressure modes, stabilization

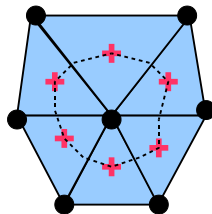
## finite volumes

UnTRIM



- elevation at circumcenters
- ⊕ normal velocity at mid edges

FVCOM



- elevation at nodes
- ⊕ velocity at baricenters

## What kind of time stepping provides stable and efficient algorithms?

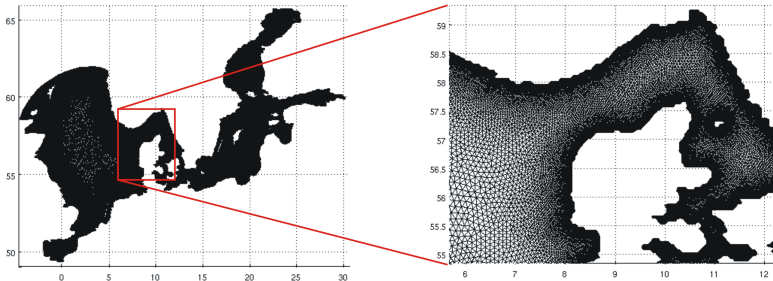
- ▶ semi-implicit: bigger time steps, but matrix inversion (needs a solver)
- ▶ explicit: small time steps for stability
- ▶ Runge-Kutta: more iterations per time step
- ▶ Adam-Bashforth: more storage

	Leap frog	Runge-Kutta	Adam-Bashforth	semi-implicit
P1P1				x
NC	x		(x)	x
FV		x	x	x

# Model intercomparison in the North Sea

## M2 tidal wave

- ▶ open boundary conditions: TPXO6.2 (OTPS Egbert et al)
- ▶ closed boundary condition: free-slip
- ▶ bathymetry: GEBCO 1min

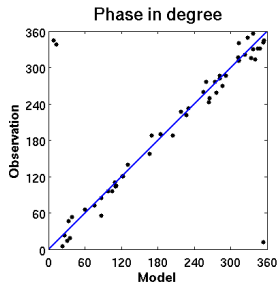
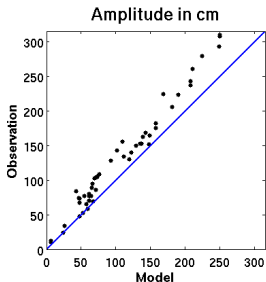


## Results in the North Sea

Under the limitations

- ▶ no wetting and drying
- ▶ minimal depth of 5m
- ▶ topography not tuned
- ▶ constant bottom friction

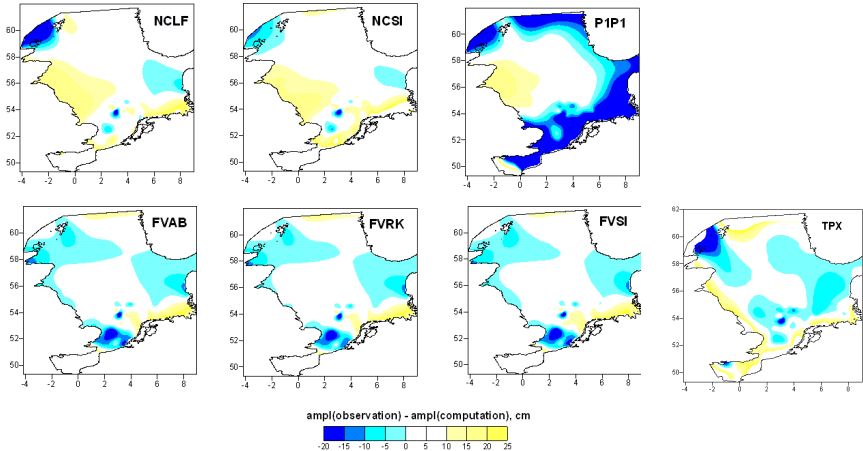
most of the codes give good results





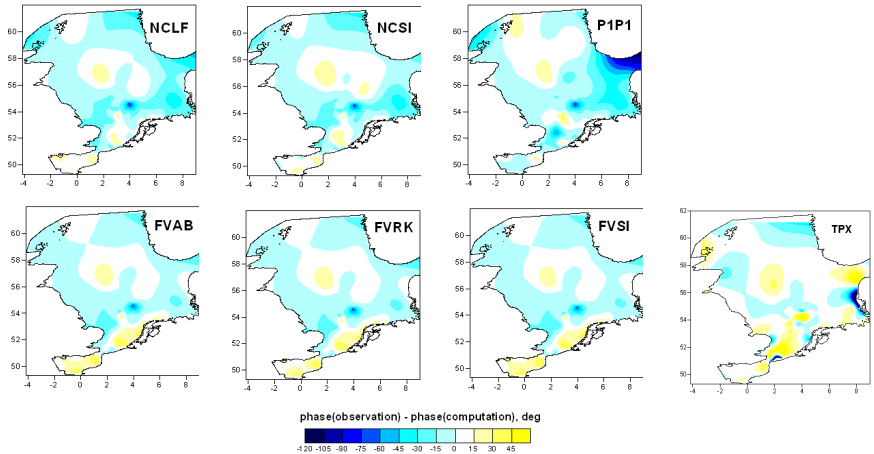
# Error - spatial distribution

## Amplitude

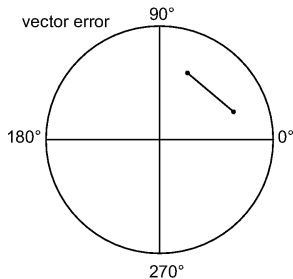
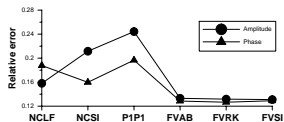
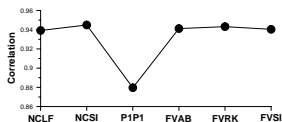
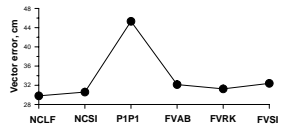


# Error - spatial distribution

## Phase

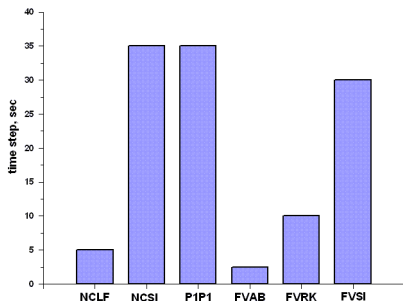
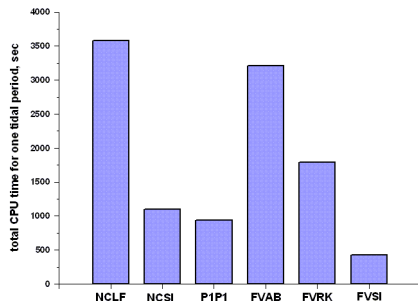


## Error statistics



$$\text{vector error} = \frac{1}{N} \sum_{n=1}^N [(A_* \cos \varphi_* - A \cos \varphi)^2 + (A_* \sin \varphi_* - A \sin \varphi)^2]^{1/2}$$

## Computational cost



- ▶ IBM p655 cluster (5 nodes with 8 CPUs each)
- ▶ use of 1 CPU of a compute node (Power4+ system (1.7GHz) with 16 GByte Ram)
- ▶ size of the mesh
  - ▶ Number of nodes = 121699
  - ▶ Number of edges = 355589
  - ▶ Number of volumes = 233872

## Conclusions

- ▶ all models give reasonable results
- ▶ semi-implicit codes are faster with same accuracy

## Outlook

- ▶ adjoint model via automatic differentiation
- ▶ sensitivity of bottom topography and bottom friction
- ▶ optimization of parameters, initial and boundary condition
- ▶ wetting & drying (done for NC, under testing)
- ▶ astronomical tides (important for the Baltic Sea)

## Questions

- ▶ How does the energy balance between M2, M4 and M6 look like?
- ▶ What is the accuracy of the energy equation?
- ▶ What is the influence of numerical diffusion?
- ▶ What are good wetting & drying schemes to give correct tidal dissipation rates?