Double diffusion in Astrophysics and Oceanography

Metstroem presentation Hannover

F. Zaussinger (MPA) and T. Zweigle (AWI)

23. July 2008

Modelling double diffusive convection

2 gradients:

- Astro: $\nabla := \frac{\partial \ln T}{\partial \ln P}, \ \nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$

2 diffusivities:

- Astro: $\kappa_T, \ \kappa_Y, \ \nu$

Modelling double diffusive convection

2 gradients:

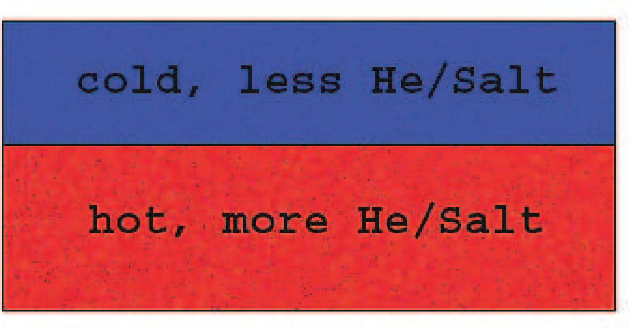
- Astro: $\nabla := \frac{\partial \ln T}{\partial \ln P}, \; \nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$
- Ocean: $\nabla T = \frac{\partial T}{\partial z}, \; \nabla S = \frac{\partial S}{\partial z}$

2 diffusivities:

- Astro: κ_T, κ_Y, ν
- Ocean: κ_T, κ_S, ν

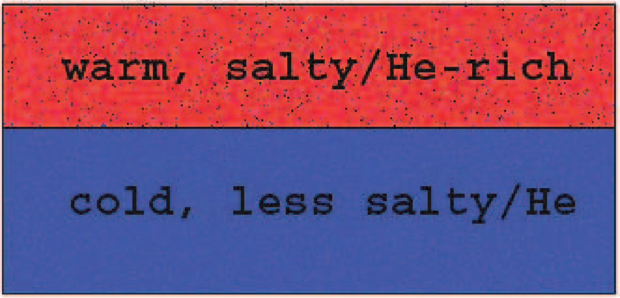
Structure of interest in Astrophysics

Semiconvection:
He/Salt stabilizes, T destabilizes



Structure of interest in Oceanography

Saltfingers:
T stabilizes, He/Salt destabilizes



Comparable? YES!

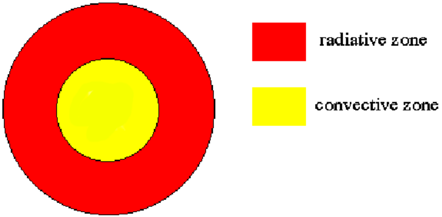
- Plasma and water are both fluids. Using (nearly) the same equations.
- $\frac{Le}{Pr} \approx \frac{1}{100}$
- 2 gradients (He/T and Salt/T) for double diffusive convection.

Comparable? YES!

- Plasma and water are both fluids. Using (nearly) the same equations.
- $\frac{Le}{Pr} \approx \frac{1}{100}$
- 2 gradients (He/T and Salt/T) for double diffusive convection.
- saltfingers in stars: off center burning
- semiconvection in ocean: close to melting icebergs

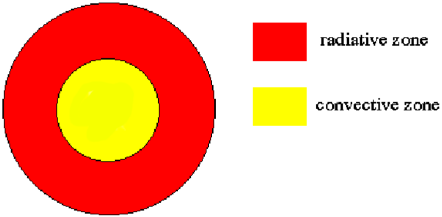
ASTROPHYSICS – SEMICONVECTION

Structure of massive stars



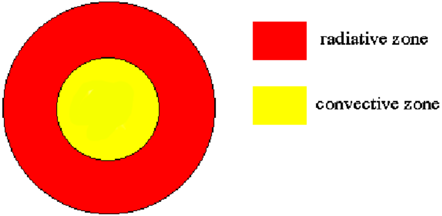
- $M \geq 9M_{\odot}$, $9 * 2 * 10^{33} g \sim 3Mio * M_{earth}$

Structure of massive stars



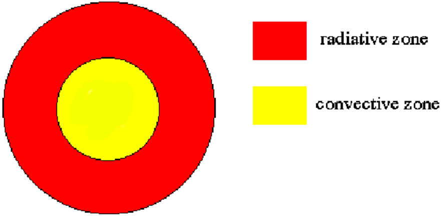
- $M \geq 9M_\odot$, $9 * 2 * 10^{33}g \sim 3Mio * M_{earth}$
- convective core, radiative envelope

Structure of massive stars



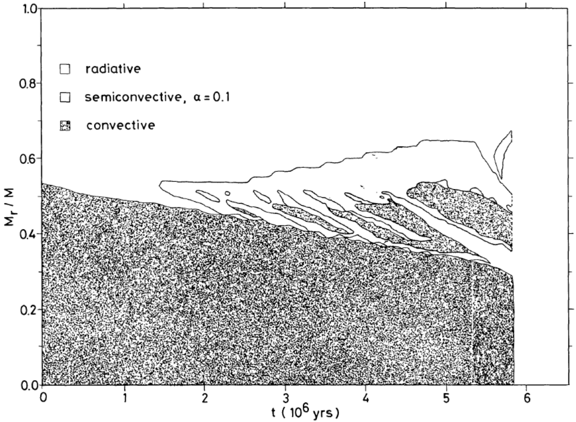
- $M \geq 9M_\odot$, $9 * 2 * 10^{33} g \sim 3 Mio * M_{earth}$
- convective core, radiative envelope
- central hydrogen burning ($T \sim 40 MioK$)

Structure of massive stars



- $M \geq 9M_{\odot}$, $9 * 2 * 10^{33} g \sim 3 Mio * M_{earth}$
- convective core, radiative envelope
- central hydrogen burning ($T \sim 40 MioK$)
- semiconvection zone is a layer "between" core and envelope
- SCZ is left behind by shrinking core (during evolution)

Semiconvection Zone in massive stars



see Langer et al

Stability and instability criterions

- $\nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$, $\nabla_{ad} := \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad}$, $\nabla := \frac{\partial \ln T}{\partial \ln P}$

Stability and instability criterions

- $\nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$, $\nabla_{ad} := \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad}$, $\nabla := \frac{\partial \ln T}{\partial \ln P}$
- $R_\mu = \frac{\nabla_\mu}{\nabla - \nabla_{ad}}$ stability parameter

Stability and instability criterions

- $\nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$, $\nabla_{ad} := (\frac{\partial \ln T}{\partial \ln P})_{ad}$, $\nabla := \frac{\partial \ln T}{\partial \ln P}$
- $R_\mu = \frac{\nabla_\mu}{\nabla - \nabla_{ad}}$ stability parameter
- $N^2 = gH_p^{-1}(\nabla_\mu - (\nabla - \nabla_{ad}))$ Brunt-Väisälä frequency

Stability and instability criterions

- $\nabla_\mu := \frac{\partial \ln \mu}{\partial \ln P}$, $\nabla_{ad} := (\frac{\partial \ln T}{\partial \ln P})_{ad}$, $\nabla := \frac{\partial \ln T}{\partial \ln P}$
- $R_\mu = \frac{\nabla_\mu}{\nabla - \nabla_{ad}}$ stability parameter
- $N^2 = gH_p^{-1}(\nabla_\mu - (\nabla - \nabla_{ad}))$ Brunt-Väisälä frequency
-

semiconvection, if:	$\nabla - \nabla_{ad} > 0$	$\nabla_\mu > 0$	$R_\mu > 0$
stable:	$N^2 > 0$	$\nabla_\mu > \nabla - \nabla_{ad}$	$R_\mu > 1$
unstable:	$N^2 < 0$	$\nabla_\mu < \nabla - \nabla_{ad}$	$R_\mu < 1$

Parameters

- Prandtl number: $Pr = \sigma = \frac{\nu}{\kappa_T} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$
- water ~ 7 , mercury ~ 0.015 , in stars $\ll 10^{-6}$

Parameters

- Prandtl number: $Pr = \sigma = \frac{\nu}{\kappa_T} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$
- water ~ 7 , mercury ~ 0.015 , in stars $\ll 10^{-6}$
- small Pr = heat diffuses very quickly compared to the velocity

Parameters

- Prandtl number: $Pr = \sigma = \frac{\nu}{\kappa_T} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$
- water ~ 7 , mercury ~ 0.015 , in stars $\ll 10^{-6}$
- small Pr = heat diffuses very quickly compared to the velocity
- Lewis number: $Le = \tau = \frac{\kappa_{He}}{\kappa_T} = \frac{\text{mass diffusivity}}{\text{thermal diffusivity}}$

Parameters

- Prandtl number: $Pr = \sigma = \frac{\nu}{\kappa_T} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$
- water ~ 7 , mercury ~ 0.015 , in stars $\ll 10^{-6}$
- small Pr = heat diffuses very quickly compared to the velocity
- Lewis number: $Le = \tau = \frac{\kappa_{He}}{\kappa_T} = \frac{\text{mass diffusivity}}{\text{thermal diffusivity}}$
- in stars $Le \ll Pr \ll 1$, $\frac{Le}{Pr} \approx \frac{1}{100}$ and $\kappa_{He} \ll \nu \ll \kappa_T$

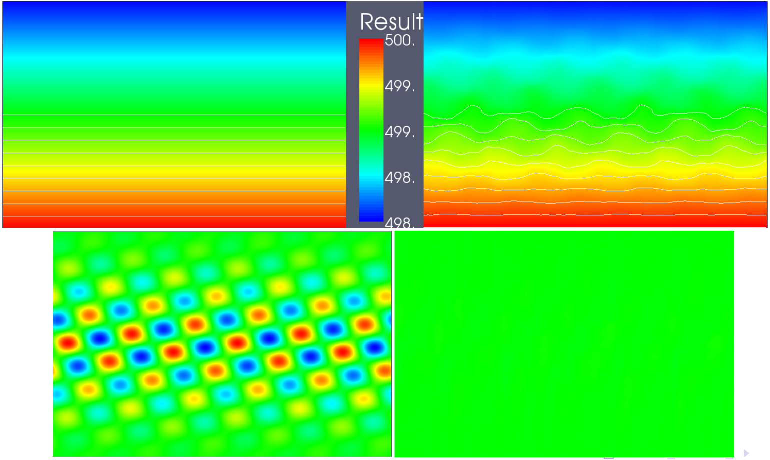
Simulation 1

- Grid: 160 vertical x 240 horizontal $\cong$ 1500km x 2250km $\cong \triangle x = 9,43km$, $\triangle t = 5 * 10^{-3} scrt$
- $Pr = 0.05$, $Le = 0.04$
- total simulation time: 33min

Simulation 1

- Grid: 160 vertical x 240 horizontal $\cong$ 1500km x 2250km $\cong \triangle x = 9,43 km$, $\triangle t = 5 * 10^{-3} scrt$
- $Pr = 0.05$, $Le = 0.04$
- total simulation time: 33min
- stable
- **LES**

Simulation 1 - stable – mass-fraction He and x-momentum



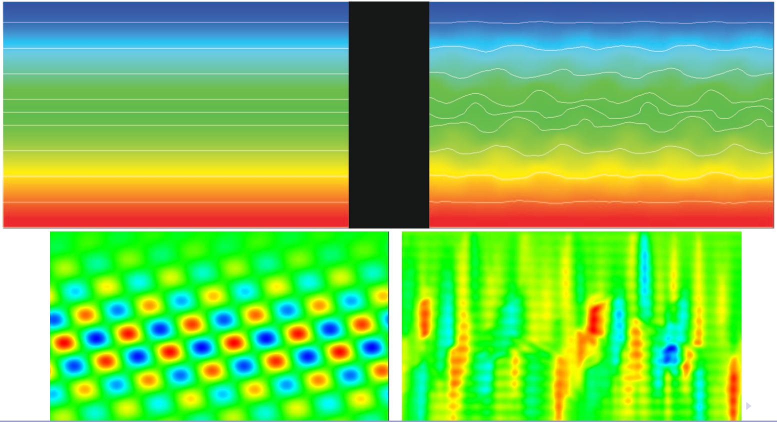
Simulation 2 - unstable

- Grid: 160 vertical x 240 horizontal $\cong$ 1500km x 2250km $\cong \triangle x = 9,43 km$, $\triangle t = 5 * 10^{-3} scrt$
- $Pr = 0.25$, $Le = 0.0175$
- total simulation time: 33min

Simulation 2 - unstable

- Grid: 160 vertical x 240 horizontal $\cong$ 1500km x 2250km $\cong \triangle x = 9,43km,\ \triangle t = 5 * 10^{-3} scrt$
- $Pr = 0.25,\ Le = 0.0175$
- total simulation time: 33min
- unstable
- **LES**

Simulation 2 - unstable – mass-fraction He and x-momentum



OCEANOGRAPHY – SALTFINGERS

Oceanography – Saltfingers

- Thermohaline staircases are a possible result of Saltfingers.
- Double–diffusion can have an effect on large scale results.

(see Merryfield et al 1999, Journal of Phys. Oceanography)

Oceanography – Saltfingers

- Thermohaline staircases are a possible result of Saltfingers.
- Double–diffusion can have an effect on large scale results.

(see Merryfield et al 1999, Journal of Phys. Oceanography)
Saltfingering occurs e.g.:

Oceanography – Saltfingers

- Thermohaline staircases are a possible result of Saltfingers.
- Double–diffusion can have an effect on large scale results.

(see Merryfield et al 1999, Journal of Phys. Oceanography)
Saltfingering occurs e.g.:

- Mediterranean outflow into Atlantic

Oceanography – Saltfingers

- Thermohaline staircases are a possible result of Saltfingers.
- Double–diffusion can have an effect on large scale results.

(see Merryfield et al 1999, Journal of Phys. Oceanography)
Saltfingering occurs e.g.:

- Mediterranean outflow into Atlantic
- River mouthes (Po outflow into Adriatic sea)

Oceanography – Saltfingers

- Thermohaline staircases are a possible result of Saltfingers.
- Double–diffusion can have an effect on large scale results.

(see Merryfield et al 1999, Journal of Phys. Oceanography)
Saltfingering occurs e.g.:

- Mediterranean outflow into Atlantic
- River mouthes (Po outflow into Adriatic sea)
- Tropical western Atlantic

Oceanography – Saltfingers

- Thermohaline staircases are a possible result of Saltfingers.
- Double–diffusion can have an effect on large scale results.

(see Merryfield et al 1999, Journal of Phys. Oceanography)
Saltfingering occurs e.g.:

- Mediterranean outflow into Atlantic
- River mouthes (Po outflow into Adriatic sea)
- Tropical western Atlantic
- Polar regions (Semiconvection)

Ocean Model and Data

Ocean Model and Data

- $\sigma = \frac{\nu}{\kappa_T} \approx 7$ (Prandtl number)

Ocean Model and Data

- $\sigma = \frac{\nu}{\kappa_T} \approx 7$ (Prandtl number)
- $\tau_{ocean} = \frac{\kappa_T}{\kappa_S} = 100$ (Lewis number)

Ocean Model and Data

- $\sigma = \frac{\nu}{\kappa_T} \approx 7$ (Prandtl number)
- $\tau_{ocean} = \frac{\kappa_T}{\kappa_S} = 100$ (Lewis number)
- $Ra \sim 10^5$ (Rayleigh number)

Ocean Model and Data

- $\sigma = \frac{\nu}{\kappa_T} \approx 7$ (Prandtl number)
- $\tau_{ocean} = \frac{\kappa_T}{\kappa_S} = 100$ (Lewis number)
- $Ra \sim 10^5$ (Rayleigh number)
- in oceans $\tau_{ocean}^{-1} \ll \sigma, \frac{1}{\sigma * \tau_{ocean}} = \frac{1}{700}$ and $\kappa_S \ll \kappa_T < \nu$

Simulation

2D–Simulation of Saltfingers

$$\begin{array}{cc} 512 \times 1024 & \text{Gridpoints} \\ 16.5 \times 33 & cm^2 \end{array}$$

Simulation

2D–Simulation of Saltfingers

$$\begin{array}{cc} 512 \times 1024 & \text{Gridpoints} \\ 16.5 \times 33 & cm^2 \end{array}$$

- $\Delta T = 1^\circ C$, $\Delta S = 0.33$

Simulation

2D–Simulation of Saltfingers

$$\begin{array}{ll} 512 \times 1024 & \text{Gridpoints} \\ 16.5 \times 33 & cm^2 \end{array}$$

- $\Delta T = 1^\circ C$, $\Delta S = 0.33$
- $\sigma = 6.2$, $\tau_{ocean} = 100$

Simulation

2D–Simulation of Saltfingers

$$\begin{array}{cc} 512 \times 1024 & \text{Gridpoints} \\ 16.5 \times 33 & cm^2 \end{array}$$

- $\Delta T = 1^\circ C$, $\Delta S = 0.33$
- $\sigma = 6.2$, $\tau_{ocean} = 100$
- time $\approx$ 90 sec with $\Delta t = 0.001$ sec

Simulation

2D–Simulation of Saltfingers

512×1024	Gridpoints
16.5×33	cm^2

- $\Delta T = 1^\circ C$, $\Delta S = 0.33$
- $\sigma = 6.2$, $\tau_{ocean} = 100$
- time ≈ 90 sec with $\Delta t = 0.001$ sec
- **DNS**

Simulations

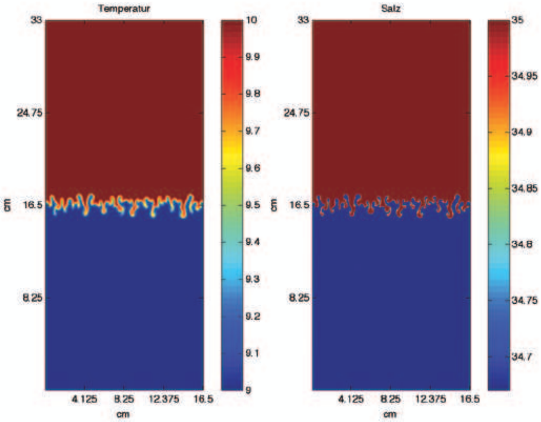


Figure: $t = 30$ sec

Simulations

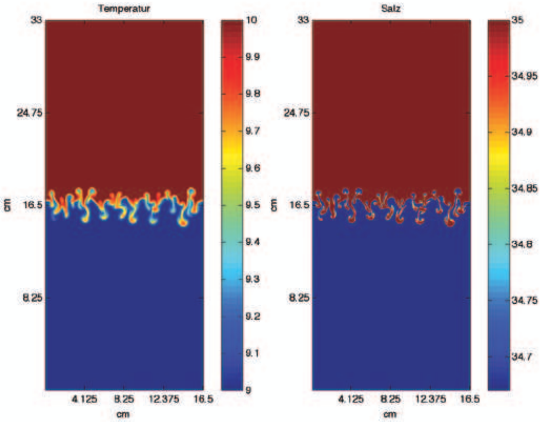


Figure: $t = 40$ sec

Simulations

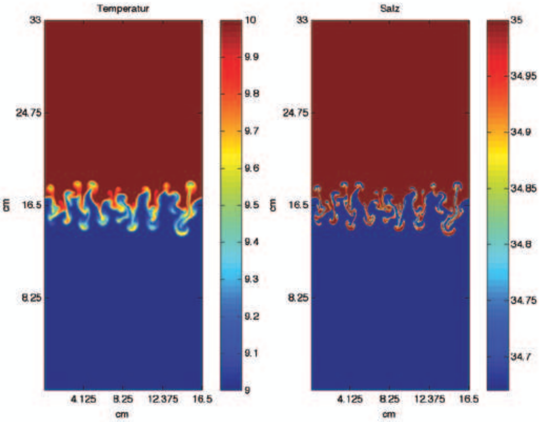


Figure: $t = 50$ sec

Simulations

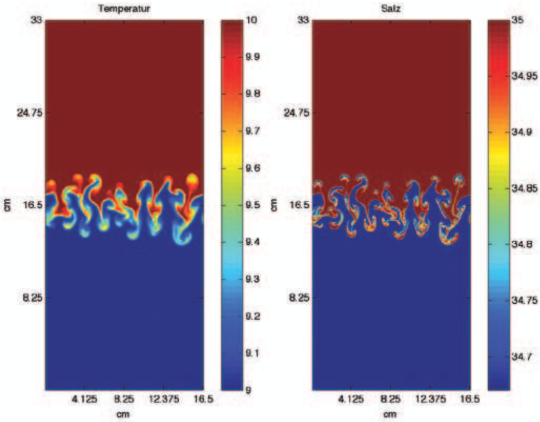


Figure: $t = 60$ sec

Running Simulation

3D–Simulation

Running Simulation

3D–Simulation

$$\begin{array}{ll} 512 \times 512 \times 512 & \text{Gridpoints} \\ 8.25 \times 8.25 \times 8.25 & cm^3 \end{array}$$

Running Simulation

3D–Simulation

$$\begin{array}{ll} 512 \times 512 \times 512 & \text{Gridpoints} \\ 8.25 \times 8.25 \times 8.25 & cm^3 \end{array}$$

- $\tau_{ocean} = 100,\ \sigma \approx 6.77$

Running Simulation

3D–Simulation

$512 \times 512 \times 512$	Gridpoints
$8.25 \times 8.25 \times 8.25$	cm^3

- $\tau_{ocean} = 100$, $\sigma \approx 6.77$
- $\Delta T = 1^\circ C$, $\Delta S = 0.17$

Running Simulation

3D–Simulation

$512 \times 512 \times 512$	Gridpoints
$8.25 \times 8.25 \times 8.25$	cm^3

- $\tau_{ocean} = 100$, $\sigma \approx 6.77$
- $\Delta T = 1^\circ C$, $\Delta S = 0.17$
- estimated time $\approx$ 80 sec with $\Delta t = 0.001$ sec

Running Simulation

3D–Simulation

$512 \times 512 \times 512$	Gridpoints
$8.25 \times 8.25 \times 8.25$	cm^3

- $\tau_{ocean} = 100$, $\sigma \approx 6.77$
- $\Delta T = 1^\circ C$, $\Delta S = 0.17$
- estimated time ≈ 80 sec with $\Delta t = 0.001$ sec
- **DNS**

Next Steps

- The intention are simulations with

$$1024 \times 1024 \times 1024 \qquad \text{Gridpoints}$$

Next Steps

- The intention are simulations with

$$1024 \times 1024 \times 1024 \qquad \text{Gridpoints}$$

- Test existing models

Next Steps

- The intention are simulations with

$$1024 \times 1024 \times 1024 \qquad \text{Gridpoints}$$

- Test existing models