# Multi-scale modelling of drop sedimentation with moment methods

SPP 1276 MetStröm: Multi-scale modelling of the population dynamics of hydrometeors with moment methods

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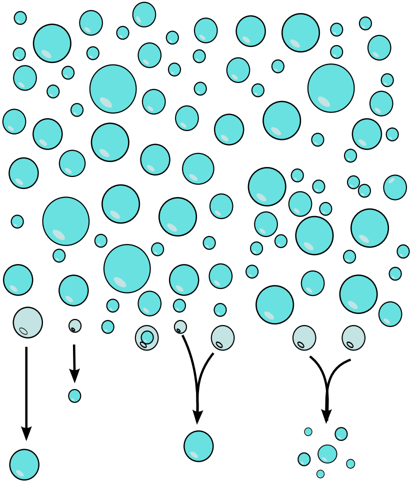
# How do we describe clouds and rain?



Variety of processes:

- sedimentation
- evaporation
- collisions

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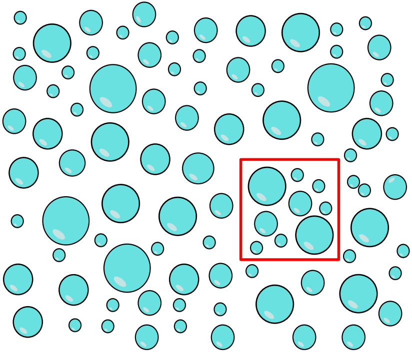
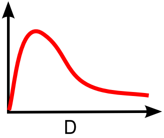


Variety of processes:

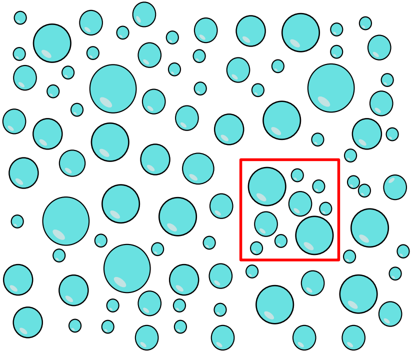
- sedimentation
- evaporation
- collisions

Description: budget equation for Spectrum $f(t, \vec{r}, D)$:

...but calculations are very costly!



# How do we describe clouds and rain?



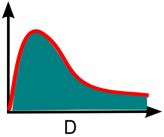
Variety of processes:

- sedimentation
- evaporation
- collisions

Description: budget equation for Spectrum $f(t, \vec{r}, D)$:
...but calculations are very costly!

Use bulk properties for description:
Moments of the spectrum

$N = M_0$: number density
$L \sim M_3$: liquid water cnt.
$Z = M_6$: radar reflectivity



$$M_k = \int_0^\infty D^k f(D)\, dD$$

...cheap, but not exact.

# Principles of the Method of Moments (MOM)

$$\boxed{\partial_t f(t, \vec{r}, D) = \sigma(f, D)}$$

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$$\boxed{\partial_t f(t,\vec{r},D) = \sigma(f,D)} \xrightarrow{\text{solve}} f(t,\vec{r},D)$$

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$$\begin{tikzcd} \boxed{\partial_t f(t,\vec{r},D) = \sigma(f,D)} \arrow[r, "\text{solve}"] & f(t,\vec{r},D) \arrow[d, dashed, "\text{integrate}"', "\text{over D}"] \\ & \mathcal{M}_j(t,\vec{r}),\ \mathcal{M}_k,\ \mathcal{M}_l,\ \ldots \end{tikzcd}$$

reference solution

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\begin{array}{l} \partial_t M_j(t,\vec{r}) \;=\; \mathscr{O}^j \\ \partial_t M_k(t,\vec{r}) \;=\; \mathscr{O}^k \\ \partial_t M_l(t,\vec{r}) \;=\; \mathscr{O}^l \end{array} & \begin{array}{c} \mathcal{M}_j(t,\vec{r}),\ \mathcal{M}_k,\ \mathcal{M}_l,\ \ldots \\ \text{reference solution} \end{array}
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\begin{aligned} \partial_t M_j(t,\vec{r}) &= \mathscr{O}^j(M_j,\ldots) \\ \partial_t M_k(t,\vec{r}) &= \mathscr{O}^k(M_j,M_k,\ldots) \\ \partial_t M_l(t,\vec{r}) &= \mathscr{O}^l(M_j,M_k,M_l,\ldots) \end{aligned} & & \begin{array}{c} \mathcal{M}_j(t,\vec{r}),\ \mathcal{M}_k,\ \mathcal{M}_l,\ \ldots \\ \text{reference solution} \end{array} \\
\begin{array}{c} f(D) = n_0\, D^{\mu}\, e^{-\lambda D} \\ \text{parameterisation assumptions} \end{array} \arrow[u] & &
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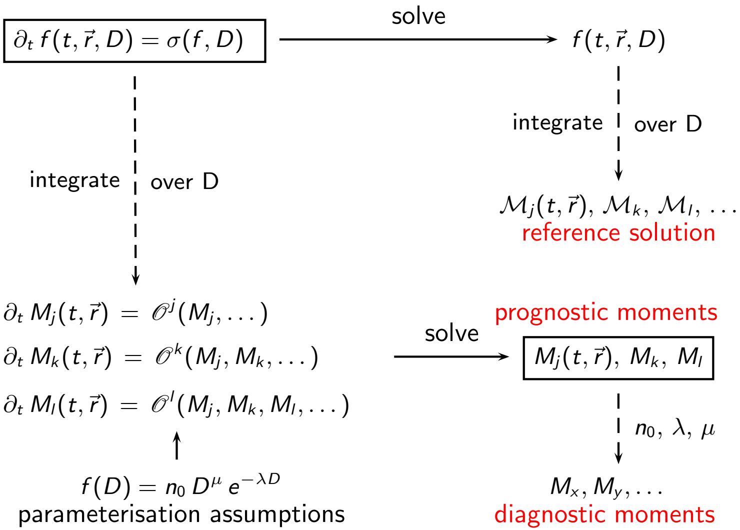


$\boxed{\partial_t f(t, \vec{r}, D) = \sigma(f, D)}$

solve

$f(t, \vec{r}, D)$

integrate over D

$\mathcal{M}_j(t, \vec{r}),\ \mathcal{M}_k,\ \mathcal{M}_l,\ \dots$

reference solution

integrate over D

$$\partial_t M_j(t, \vec{r}) = \mathscr{O}^j(M_j, \dots)$$
$$\partial_t M_k(t, \vec{r}) = \mathscr{O}^k(M_j, M_k, \dots)$$
$$\partial_t M_l(t, \vec{r}) = \mathscr{O}^l(M_j, M_k, M_l, \dots)$$

prognostic moments

solve

$\boxed{M_j(t, \vec{r}),\ M_k,\ M_l}$

$n_0,\ \lambda,\ \mu$

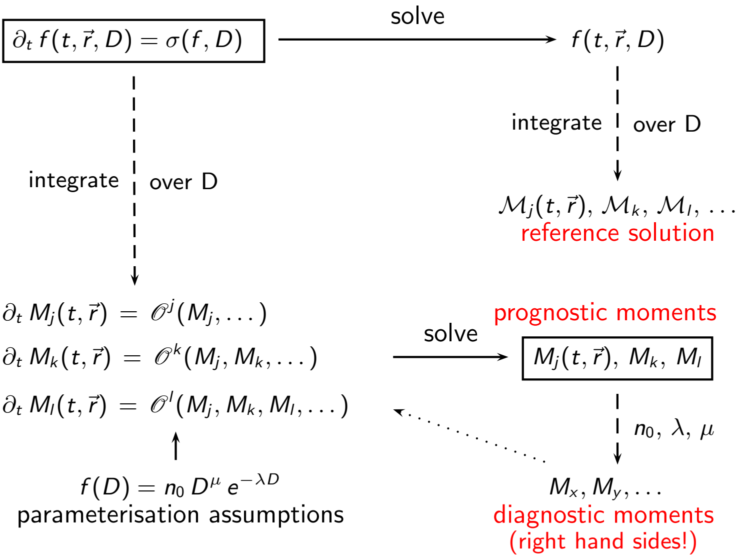
$f(D) = n_0\, D^{\mu}\, e^{-\lambda D}$

parameterisation assumptions

$M_x, M_y, \dots$

diagnostic moments

# Principles of the Method of Moments (MOM)



$\boxed{\partial_t f(t, \vec{r}, D) = \sigma(f, D)}$

solve

$f(t, \vec{r}, D)$

integrate over D

$\mathcal{M}_j(t, \vec{r}),\ \mathcal{M}_k,\ \mathcal{M}_l,\ \ldots$

reference solution

integrate over D

$$\begin{aligned} \partial_t M_j(t, \vec{r}) &= \mathscr{O}^j(M_j, \ldots) \\ \partial_t M_k(t, \vec{r}) &= \mathscr{O}^k(M_j, M_k, \ldots) \\ \partial_t M_l(t, \vec{r}) &= \mathscr{O}^l(M_j, M_k, M_l, \ldots) \end{aligned}$$

prognostic moments

solve

$\boxed{M_j(t, \vec{r}),\ M_k,\ M_l}$

$n_0,\ \lambda,\ \mu$

$M_x, M_y, \ldots$

diagnostic moments

(right hand sides!)

$f(D) = n_0\, D^{\mu}\, e^{-\lambda D}$

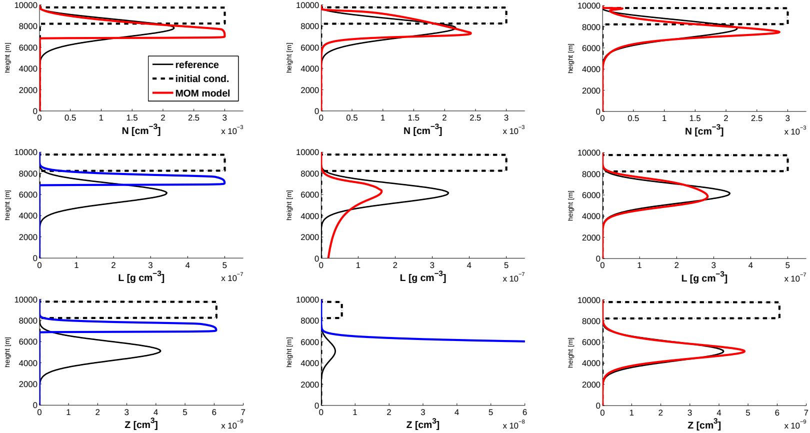
parameterisation assumptions

# How many moments should one choose?

MOM1 $f(D) = n_0 D^{\mu} e^{-\lambda D}$

MOM2 $f(D) = n_0 D^{\mu} e^{-\lambda D}$

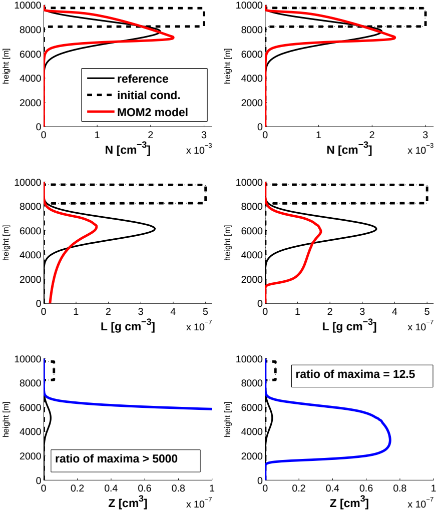
MOM3 $f(D) = n_0 D^{\mu} e^{-\lambda D}$



prognostic moment (red), diagnostic moment (blue), model time $t = 600$ s

# MOM2: Influence of maximum drop diameter

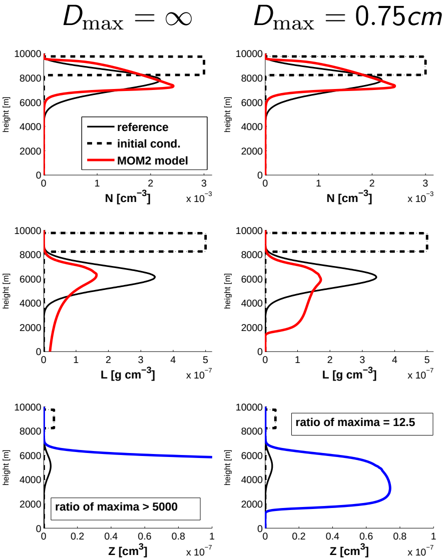
$D_{\max} = \infty$ $D_{\max} = 0.75cm$



**Recall:**

$$M_k = \int_0^{\infty} D^k f(D)\, dD$$

# MOM2: Influence of maximum drop diameter



Now: maximum drop diameter $D_{\max}$

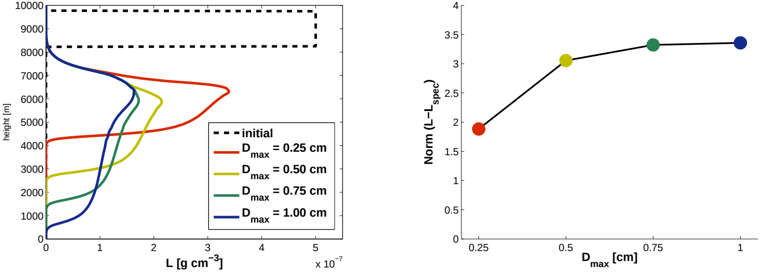
$$M_k = \int_0^{D_{\max}} D^k f(D)\, dD$$

## First impressions

- prognostic moments are 'closer' to reference solution
- better representation of diagnostic moments

prognostic moment (red), diagnostic moment (blue), model time $t = 600$ s

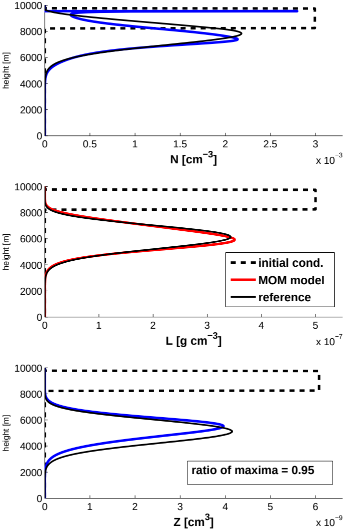
# MOM2: Sensitivity to maximum drop diameter



When $D_{\max}$ is small...

- moments are more sensitive to changes in $D_{\max}$
- prognostic moments are closer to reference solution

# MOM3: Results



- Here: prognostic moments: $M_1, \ M_2, \ M_3$

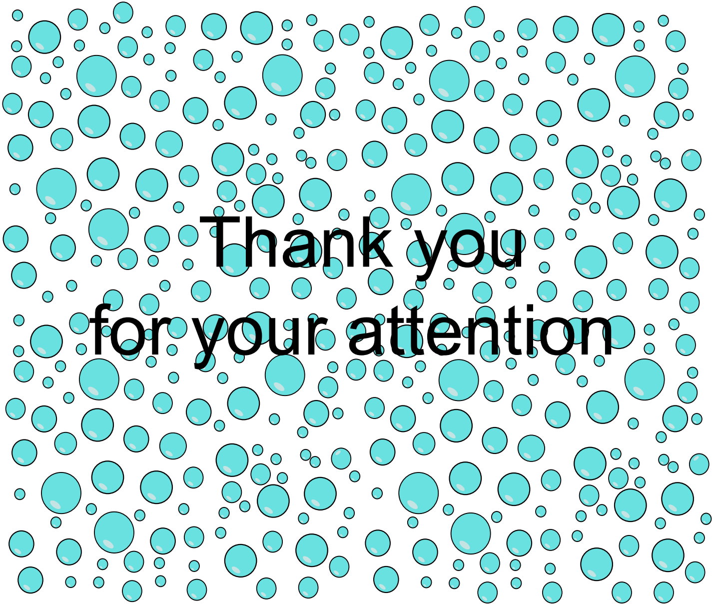
- Diagnosis of other moments
    - lower order: problematic
    - higher order: very good

MOM: prognostic moment (red),

diagnostic moment (blue), $t = 600$ s

# Conclusions

- MOM able to model the effects of microphysical processes in numerically cheap way.
- Accuracy & complexity increase with number of moments
    - MOM1: low cost, but drawbacks
    - MOM2: realistic maximum drop diameter gives better results
    - MOM3: very good results, but technical challenges
- best: forecast moments of most physical interest
- Further refinements of each technique are possible



Thank you
for your attention