A regulated localization method for ensemble-based Kalman filters

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Introduction

Localization is essential for data assimilation with ensemble-based Kalman filters in large-scale systems. Two localization methods are commonly used: Covariance localization (**CL**) and domain localization (**DL**). The former applies a localizing weight to the forecast covariance matrix while the latter splits the assimilation into local regions in which independent assimilation updates are performed. The domain localization is usually combined with a weighting of the observation error covariance matrix, denoted observation localization (**OL**). OL results in a similar localization effect to that of covariance localized filters. In order to improve the performance of domain localization with weighting of the observation errors, a regulated localization scheme is introduced. Using twin experiments with the Lorenz-96 model, it is demonstrated that the regulated localization can lead to a significant reduction of the estimation errors as well as increased stability of the assimilation process. In addition, the numerical experiments point out that the combination of covariance localization with a serial processing of observations during the analysis step can destabilize the assimilation process.

Assimilation experiments

Twin experiments are conducted using the Lorenz-96 model [7] implemented in the Parallel Data Assimilation Framework (PDAF, http://pdaf.awi.de). The ensemble has a size of 10 members. The localization functions w^{CL}, w^{OL} are chosen to be 5th-order polynomials mimicking a Gaussian function, but having compact support. In the experiments, the support radius and the forgetting factor (covariance inflation) is varied. For each pair of these parameters 10 experiments are conducted using different random numbers to generate the initial ensemble from a long state trajectory. The performance of the assimilation experiments is assessed using the time-mean RMS deviations from the true state that was used to generate the observations.

Four combinations of filter algorithms and localization methods are compared:

- LSEIK-fix: Local SEIK filter [5] using fixed OL.
- LSEIK-reg: Local SEIK filter using regulated OL.
- EnKF-sqrt: Square-root formulation of Ensemble Kalman filter (following [2]) using CL.
- EnSRF: Ensemble square-root filter with sequential processing of observations [6] using CL.

Conclusion

- OL results in a longer effective localization length scale compared to CL. The length scale increases for more accurate observations.
- A regulated localization function for OL has been introduced. For a single observation, it results in identical effective localizations for CL and OL.
- Numerical experiments show a significant improvement of the assimilation performance with regulated localization for small observation errors.
- The EnSRF method with CL showed an inferior assimilation performance. It is caused by the combination of CL with sequential processing of observations.

Effective localization of the Kalman gain

Previous studies [1–4] found that CL and OL are not equivalent. However, if the observations have only a small influence the difference induced by the localization methods is small. If the influence of the observations is larger, OL requires a smaller localization length scale and, still, can lead to inferior assimilation results than using CL.

The published findings can be explained by considering the effect of the localization on the Kalman gain. Following [1], the gain for CL is in case of a single observation:

$$\mathbf{K}^{CL} = \frac{w^{CL}}{HPH^T+\sigma_R^2}\mathbf{P}^f\mathbf{H}^T$$

For OL the gain is is:

$$\mathbf{K}^{OL} = \frac{w^{OL}}{w^{OL}HPH^T+\sigma_R^2}\mathbf{P}^f\mathbf{H}^T$$

Here, w^{CL} and w^{OL} are the localization functions applied in the CL and OL methods. For CL the localization function is a simple factor in the gain. This is not the case for OL.

The effective localization length scale in the Kalman gain is different for both methods. It depends on the relative size of the estimated state error variance (P) and observation error variance σ_R^2 as is shown in figure (1). If the observation error is particularly small, the effective localization length in OL will be much larger than that of CL.

To obtain an identical effective localization length scale in OL, a distinct weight function w^{OLR} is required. It can be derived by equating both of the gain equations shown in the left column. The calculation leads to the *regulated localization function*

$$w^{OLR} = \frac{w^{CL}\sigma_R^2}{HPH^T+\sigma_R^2}\left(1-\frac{w^{CL}HPH^T}{HPH^T+\sigma_R^2}\right)^{-1}$$

The function is always narrower than the weight function w^{CL}. It avoids the widening of the effective localization length scale for small observation errors.

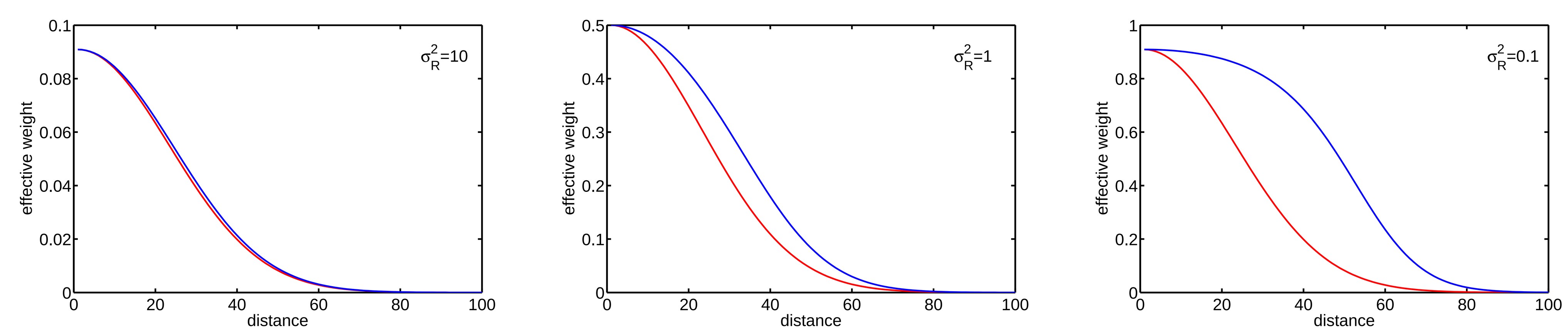

Figure 1: Effective localization functions in the Kalman gain for different observation error variances σ_R^2 and state error variance 1. (red): Weighting term for CL and for OL with regulated localization. (blue): Weighting term for OL. The effective weighting is increasingly wider for observation localization for decreasing σ_R^2.

Impact of regulated localization

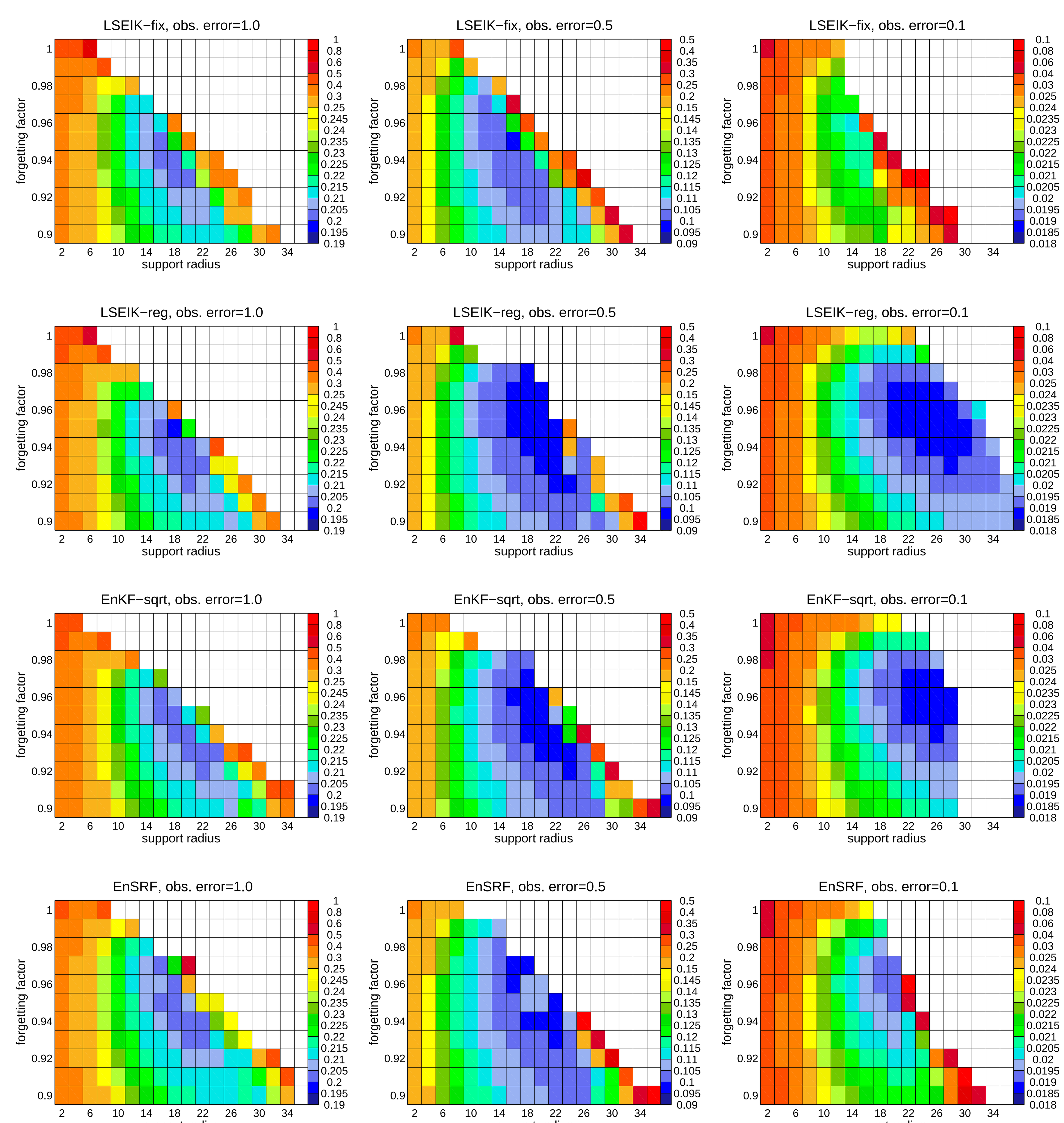

Figure 2: Time-mean RMS errors averaged over each 10 experiments.

The regulated localization (LSEIK-reg) results in a significant reduction of the errors compared to fixed-OL (LSEIK-fix), in particular for small observation errors. In addition, the parameter region with minimum errors is increased.

The EnKF-sqrt method shows errors that are very similar to those obtained with LSEIK-reg. However, EnKF-sqrt diverges in case of the smallest observation errors for long localization radii. Here, LSEIK-reg is still stable.

The EnSRF is less stable with larger errors compared to LSEIK-reg and EnKF-sqrt. This behavior is caused by the combination of CL with sequential processing of observations, which renders the update equation of the covariance matrix to be inexact.

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