

# On sequential observation processing in localized ensemble Kalman filters

## Introduction

The different variants of current ensemble square-root Kalman filters assimilate either all observations at once or perform a sequence in which batches of observations or each single observation is assimilated. The sequential observation processing in filter algorithms like the EnSRF [1] can result in computationally efficient algorithms because matrix inversions in the observation space are reduced to the inversion of single numbers.

Whitaker and Hamill [1] noted that the modification of the EnSRF for localization leads to an inconsistency of the update equation for the state error covariance matrix. Often, this inconsistency does not lead to a significant impact on the assimilation performance. However, using a simple model, we demonstrate with the localized EnSRF algorithm that the sequential observation processing can significantly deteriorate the assimilation performance under some circumstances.

We perform assimilation experiments with the Lorenz-96 model. Compared are the performances of the EnSRF with the LESTKF filter, both with localization. Assimilation experiments are performed over 50000 time steps with an ensemble of 10 states. The support radius of the localization and the inflation (forgetting factor) are varied.

For the LESTKF the regulated localization weight function [3] is used. In [3] it was shown that this method results in equal effective localization lengths for a single observation for covariance localization and observation localization.

The filter algorithms and the Lorenz96 model are implemented in the Parallel Data assimilation Framework (PDAF, [5, 6], <http://pdaf.awi.de>).

## Assimilation Experiments

### EnSRF

- Ensemble square-root filter [1]
- Assimilate an observation vector as a sequence of single observations
- Localize with state error covariance matrix ("covariance localization")

### LESTKF

- Error subspace transform Kalman filter [2]
- Assimilate full observation vector at once
- Perform local analysis with observation weights computed from regulated localization [3]

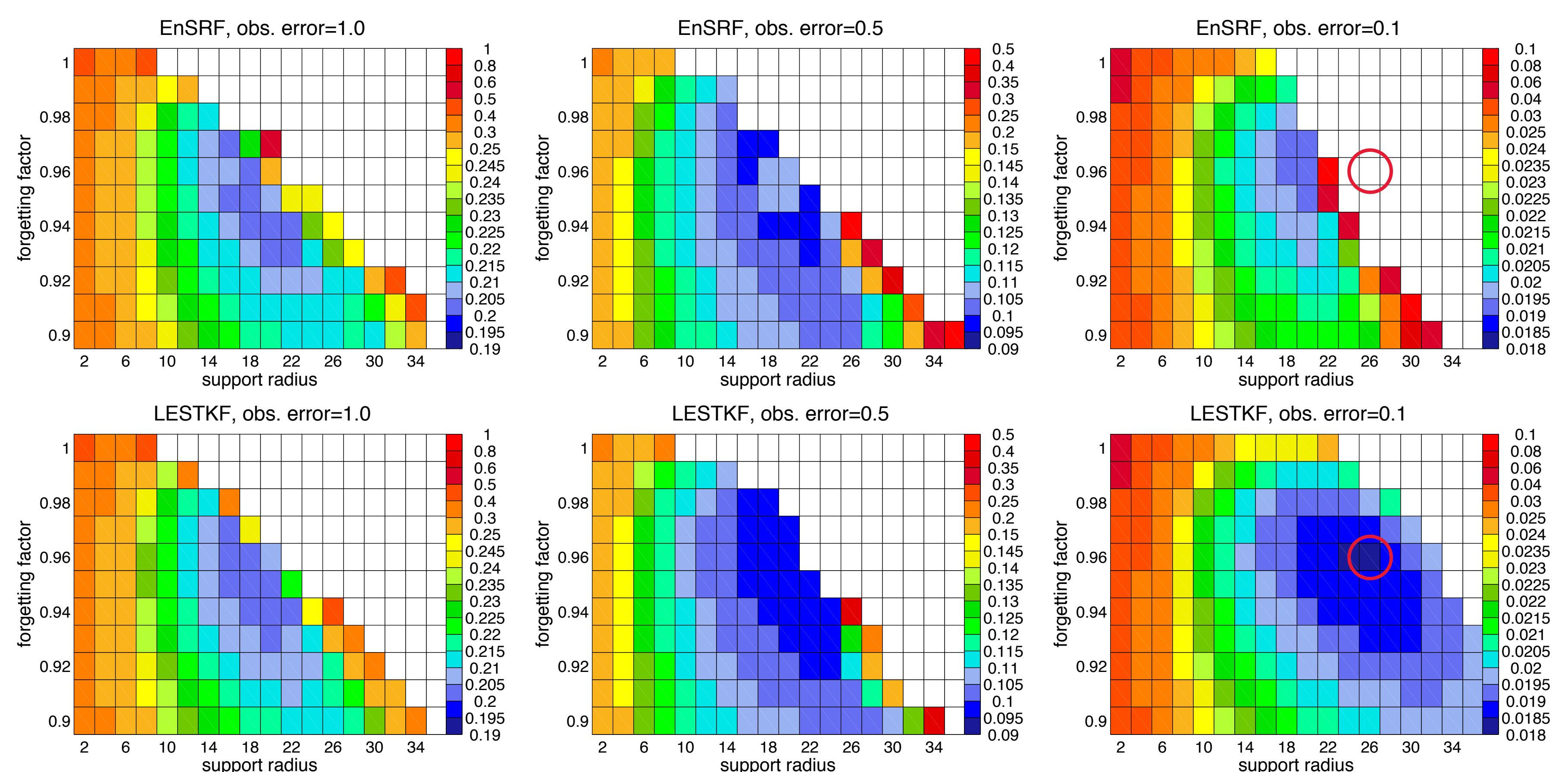
## Experiments

### Time-mean RMS errors

The figure compares mean RMS errors for different localization support radii and forgetting factors. The error in the observations is reduced from left to right. In white fields, the filter diverges.

For decreasing observation error, the difference in RMS errors between both filters increases. The errors of the EnSRF are larger than for the LESTKF. Also, the region of filter convergence is smaller for the EnSRF than for LESTKF.

To get an idea about the reason for the different performance, we focus on the optimal configuration for LESTKF: observation error 0.1; support radius: 26 grid points; forgetting factor 0.96 (see circle).

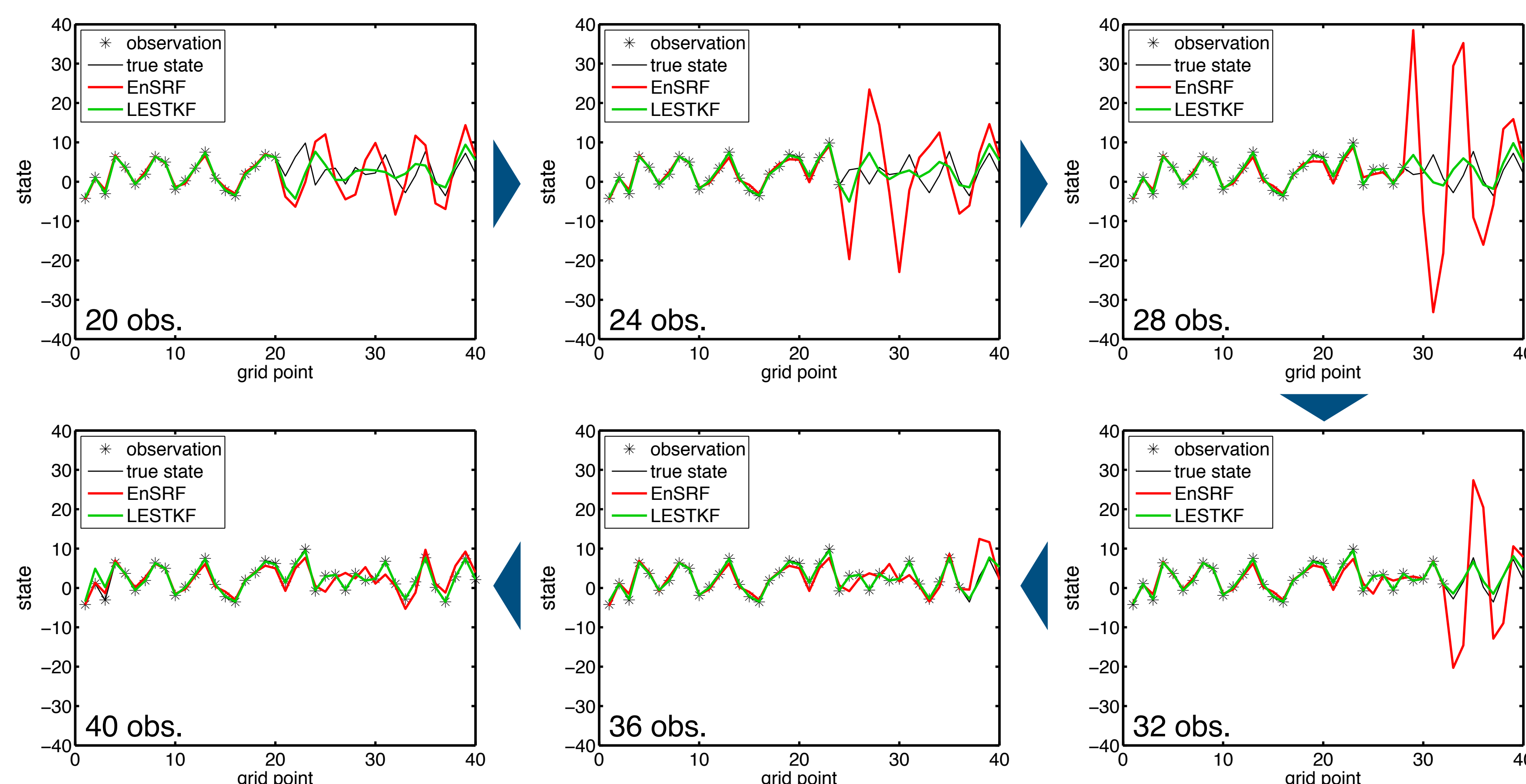
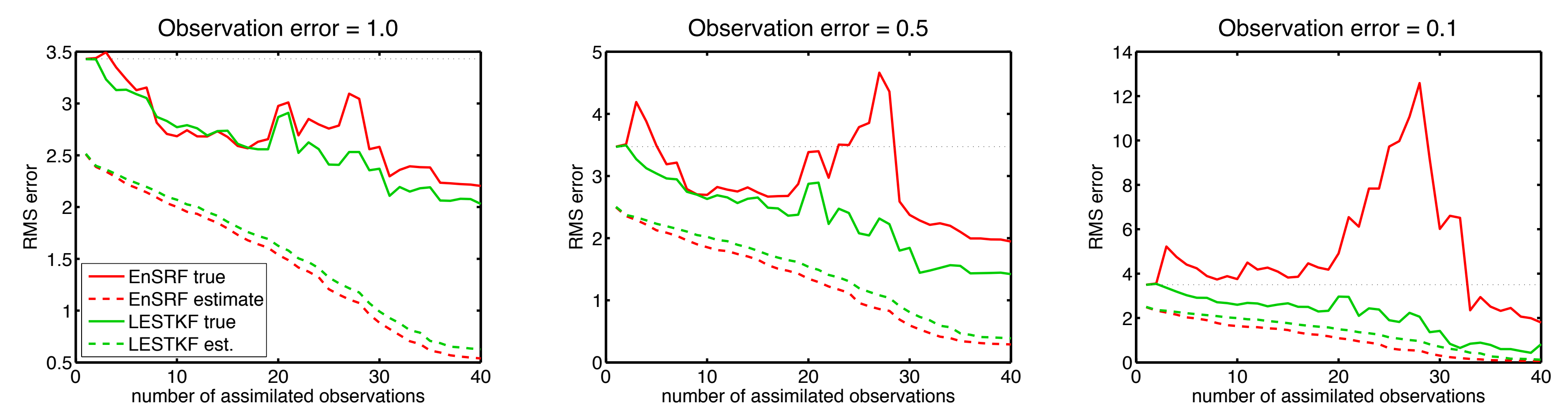


### RMS errors at first analysis time

We perform a series of experiments varying the number of observations. The EnSRF realizes each of the state estimates in its analysis sequence. The true and estimated errors at the first analysis time are different for both filters.

For the two smaller observation errors, the intermediate state estimates can have a larger RMS error than without assimilating any data. After all observations are assimilated, the state estimate from EnSRF is still worse than for LESTKF.

Thus, for a strong impact of the assimilation, the sequential observation processing in the local EnSRF can have a large deteriorating influence.



### State estimates

#### for different numbers of observations

To illustrate the reason for the large RMS errors for observation error 0.1, the state estimates are shown when different numbers of observations are assimilated. In case of the EnSRF these estimates are realized in the assimilation sequence.

**20 observations:** The wave is well estimated where observations are present. In the other half of the domain, the estimates of both filters are similar.

**24-32 observations:** The estimated wave from EnSRF shows erroneously strong oscillations in the region where no observation are assimilated yet. In contrast, the estimate of the LESTKF is still of the correct magnitude.

**36-40 observations:** The strong oscillations are finally damped. The final estimate of the EnSRF show larger errors than that of the LESTKF.

## References

- [1] Whitaker, J. S. and T. M. Hamill (2002). Ensemble data assimilation without perturbed observations. *Mon. Wea. Rev.* 130, 1913–1927
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