An adaptive error subspace method for ensemble-based Kalman filters

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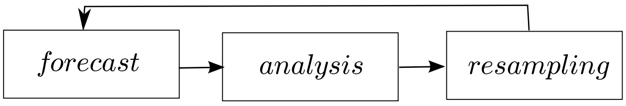
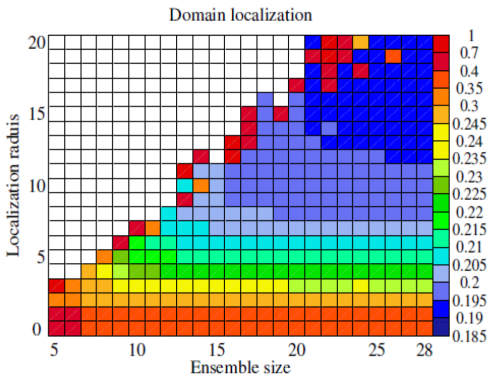
Idea

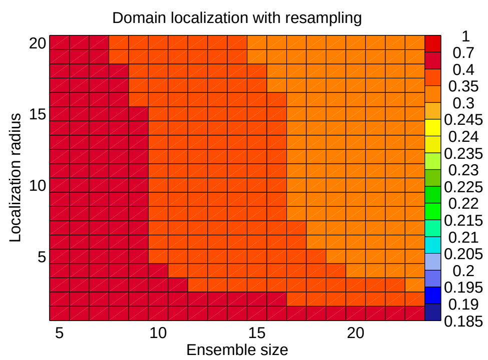
The idea of resampling is applied to the Local Ensemble Transform Kalman filter (LETKF). The resampling method was introduced by Song et.al. [1] for the Singular Evolutive Extended Kalman (SEEK) filter. In the resampling step, the ensemble member that contributes the least information to the error subspace is replaced by a newly created ensemble member in each analysis step. Based on the Lorenz-96 model it is tested whether the use of resampling is helpful in the LETKF.



The new member is created by weighting the analysis and the observation with a Kalman matrix, that uses a background covariance matrix from a long run of the model. The idea is that the new ensemble error that belongs to the new ensemble member can enrich the error subspace with a new suitable direction. Experiments are made with the Lorenz-96 model and the Parallel Data Assimilation Framework (PDAF, http://pdaf.awi.de).



MTRMSE for LETKF over 5000 time steps (from [2])



MTRMSE for LETKF over 5000 time steps with resampling

Filter Equations

ETKF:

ensemble $\mathbf{X}_i^a = [\mathbf{x}_i^{a(1)}, ..., \mathbf{x}_i^{a(r)}]$

forecast $\mathbf{x}_i^{f(i)} = \mathbf{M}_{i-1}\mathbf{x}_{i-1}^{a(j)} \ \forall j = 1, ..., r$

$\mathbf{P}_i^f = \frac{1}{r-1}\mathbf{X}_i'^f \ (\mathbf{X}_i'^f)^T$

analysis $\hat{\mathbf{K}}_i = \mathbf{P}_i^a \mathbf{H}_i^T \mathbf{R}_i^{-1}$

$\bar{\mathbf{x}}_i^a = \bar{\mathbf{x}}_i^f + \hat{\mathbf{K}}_i(\mathbf{y}_i - \mathbf{H}_i\bar{\mathbf{x}}_i^f)$

Resampling: (replace member j^*)

$$j^* := \underset{j}{\operatorname{argmin}} ||\mathbf{x}_i'^{a(j)}||$$

$$\mathbf{x}_i'^{a(j^*)} = \mathbf{K}^{res} \cdot (\mathbf{y}_i - \mathbf{H}_i\bar{\mathbf{x}}_i^a)$$

$$\mathbf{K}_i^{res} = \mathbf{B}\mathbf{H}_i^T(\mathbf{H}_i\mathbf{B}\mathbf{H}_i^T + \mathbf{R}_i)^{-1}$$

with a stationary covariance matrix $\mathbf{B}$ from an initializing run with a huge ensemble.

Assimilation experiments

The filter is tested with a 40-dimensional Lorenz-96 model in a twin experiment. A true state is generated over 5000 time steps after a spin-up of 1000 steps.

The whole state is observed at each time. A normally distributed error with standard deviation $\sigma_r = 1$ is added to the true state in order to generate the observations. The covariance matrix of the observations $\mathbf{R}$ is diagonal. The root mean square error averaged over the assimilation time and repetitions is used to evaluate the assimilation performance. It is denoted as MTRMSE. If the error is bigger than $\sigma_r = 1$, the filter diverges.

Evaluation

Without resampling, half of the filter experiments converge. To each specific ensemble size there is an optimal localization radius. The minimal value of the MTRMSE is 0.19. With resampling all cases converge. But the MTRMSE is always between 0.55 and 0.3. So the analysis can be improved by this type of resampling (no divergence), but it is not better than the best cases without resampling.

Next steps

An alternative criterion to select the ensemble member to be replaced can be examined. An idea is that the $\mathbf{x}_i'^{a(j)}$ should be replaced by the $\mathbf{H}_i^{-1}\mathbf{d}_i^a$ that is most orthogonal to $\mathbf{d}_i^a$. The presumption is that this is exactly the error that has been missing to represent the error subspace.

$$j^* := \underset{j}{\operatorname{argmin}} < \mathbf{H}\mathbf{x}_i'^{a(j)}, \mathbf{d}_i^a >$$

$$\mathbf{x}_i'^{a(j^*)} = \mathbf{H}_i^{-1}\mathbf{d}_i^a$$

References

[1] Song, H., I. Hoteit, B. D. Cornuelle, and A. C. Subramanian, 2010: An adaptive approach to mitigate background covariance limitations in the ensemble Kalman filter. Mon. Wea. Rev., 138, 2825-2845.

[2] Kirchgessner, P., Nerger, L., Bunse-Gerstner, A. (2014) On the choice of an optimal localization radius in ensemble Kalman filter methods. Monthly Weather Review, accepted, doi:10.1175/MWR-D-13-00246.1