A Data Assimilation study with the Kalman Filter on a finite element ocean model

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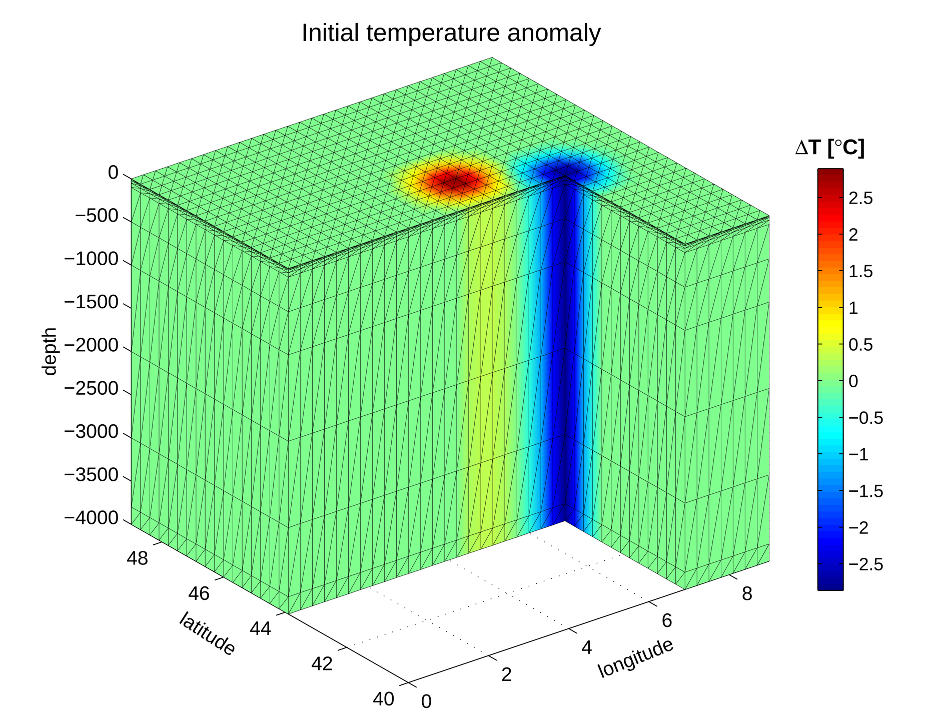
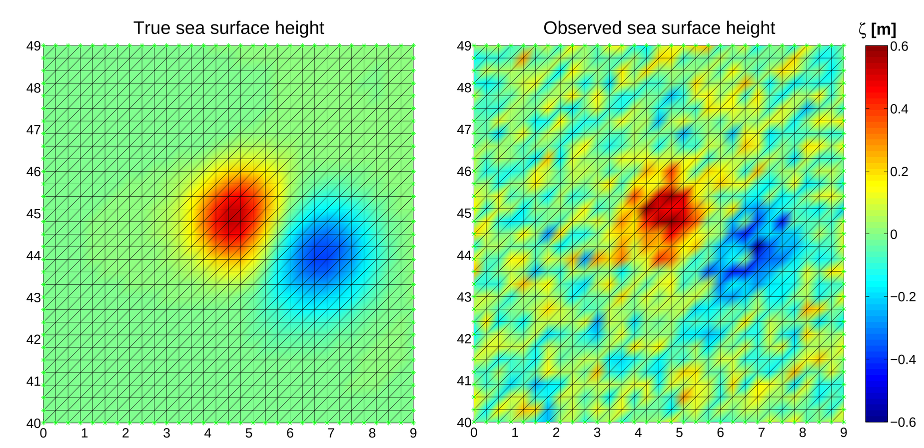
The Problem

The main objective of this work is to study the capabilities of data assimilation schemes based on the Kalman filter applied to a finite element ocean model. The assimilation system has been implemented in FENA (Finite Element Model for the North Atlantic), which has been developed recently at the Alfred-Wegener Institute. Twin experiments are performed to assimilate synthetic observations of the sea surface height with the model used in a simplified configuration. Using several experiments with the Ensemble Kalman Filter (EnKF), the Singular Evolutive Interpolated Kalman (SEIK) filter, and the Singular Evolutive Extended Kalman (SEEK) filter the capabilities of the algorithms are assessed and their feasibility for application on more realistic configurations of FENA is discussed.

FENA is based on the primitive equations discretized on an unstructured grid with variable meshsize. It is designed to study the thermohaline circulation on basin to global scales for periods of up to a century. FENA uses a tetrahedral spatial discretization, backward Euler time stepping, and approximates the model fields by linear functions on elements.
The assimilation experiments employ a simplified configuration with a rectangular box geometry, linear density stratification and linear equation of state. Further, convection is neglected. The box occupies an area of 9 by 9 degrees centered at 44.5° N and has a depth of 4000 m. It is discretized with 11 vertical levels and a horizontal grid of 31 by 31 points.
The process studied is the propagation of interacting baroclinic Rossby waves. The waves are initialized with two horizontally localized columnar temperature anomalies. The anomalies become deformed as they propagate westward, and tilt towards each other via the induced velocity field (a negative spot produces counter-clockwise rotation in upper layers and clockwise rotation in lower layers). This introduces nonlinearity which is required to test the performance of the filter algorithms.



Cut into the model domain showing the initial temperature anomalies and the tetrehedral finite element discretization.



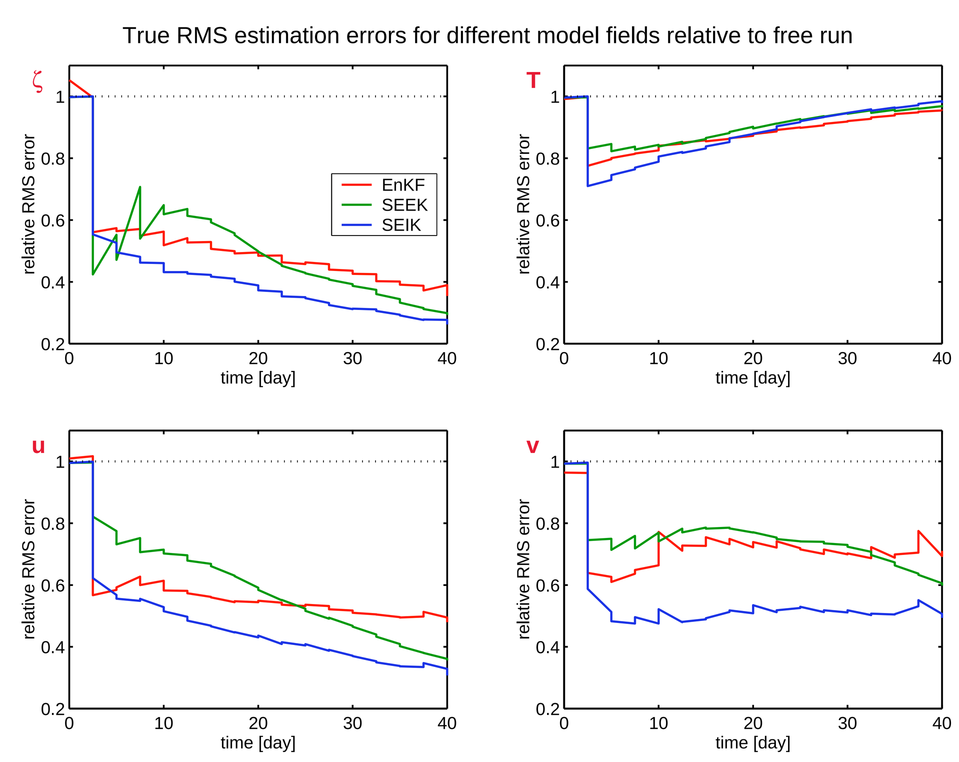
True sea surface height and observed sea surface height at begin of assimilation period.

Filter Configuration

We performed twin experiments with synthetic observations generated by adding Gaussian noise of constant variance to the sea surface height of a model run over 45 days. With this, the relative noise amplitude increases during the assimilation period to about the same level of the signal amplitude itself.
Initialization of the assimilation experiments was performed with a covariance matrix computed from the state sequences of 28 simulations using different initial locations of the temperature anomalies. The initial state estimate was chosen as the mean state of these model runs.
The experiments were conducted over a period of 40 days with an interval of 2.5 days between subsequent analysis. The observations were used with an offset of 5 days.
To account for model error we applied a wind forcing field with stochastic amplitude to the ensemble integrations performed in the EnKF and SEIK filters. Each ensemble member was forced by a different wind field which was initialized once after each analysis. For comparability, the SEEK filter was used without a forgetting factor, since it could be applied to all three filters. Thus the filtering with SEEK is performed without consideration of model error.
In the experiments the prognostic state variables are the zonal and meridional velocity components **u**, **v**, the temperature field **T**, and the sea surface height ζ. This amounts to a state dimension of $n = 32674$. The dimension of the observations vector was $m = 961$ at each analysis time.

To assess the filter performances we compare results for assimilation experiments in which all filters need to perform the same amount of model evaluations. With this all three filters need nearly the same computing time, since the forecast phase of the experiments takes more than 99% of the total execution time. To achieve this, the rank r used in the SEEK and SEIK experiments was one less than the ensemble size N of the EnKF. Below filter results for $N = 10$ and $N = 100$ are compared.

N = 10

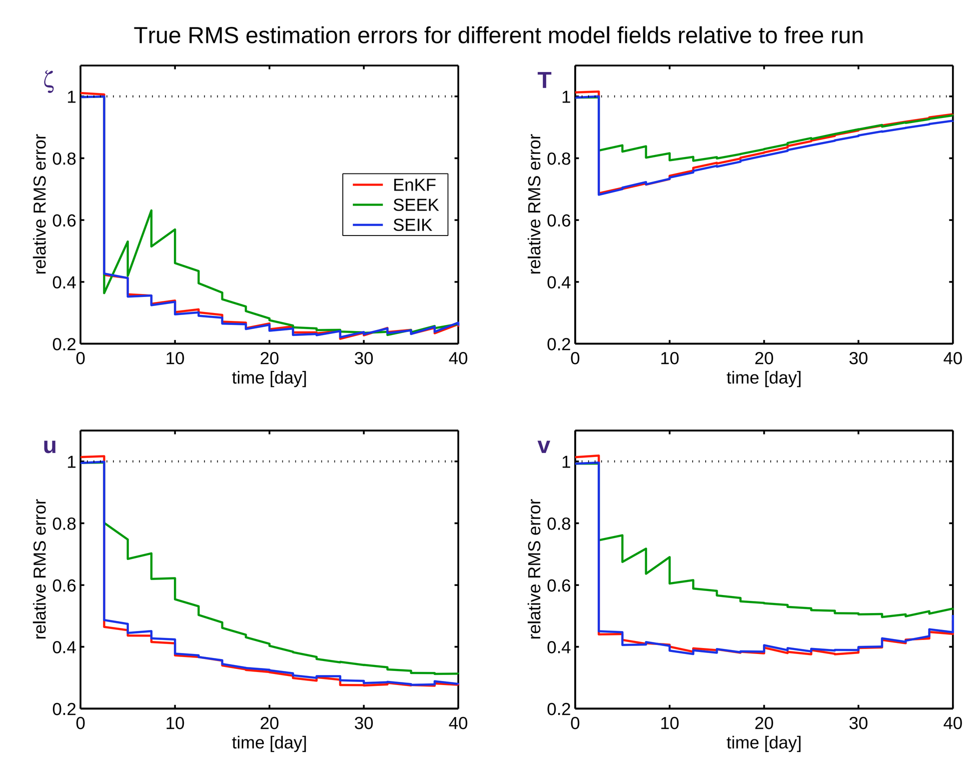


RMS estimation errors for the four model fields for assimilation with an ensemble size of $N = 10$. The RMS values are scaled by the RMS deviation of a simulation without assimilation from the true state fields.

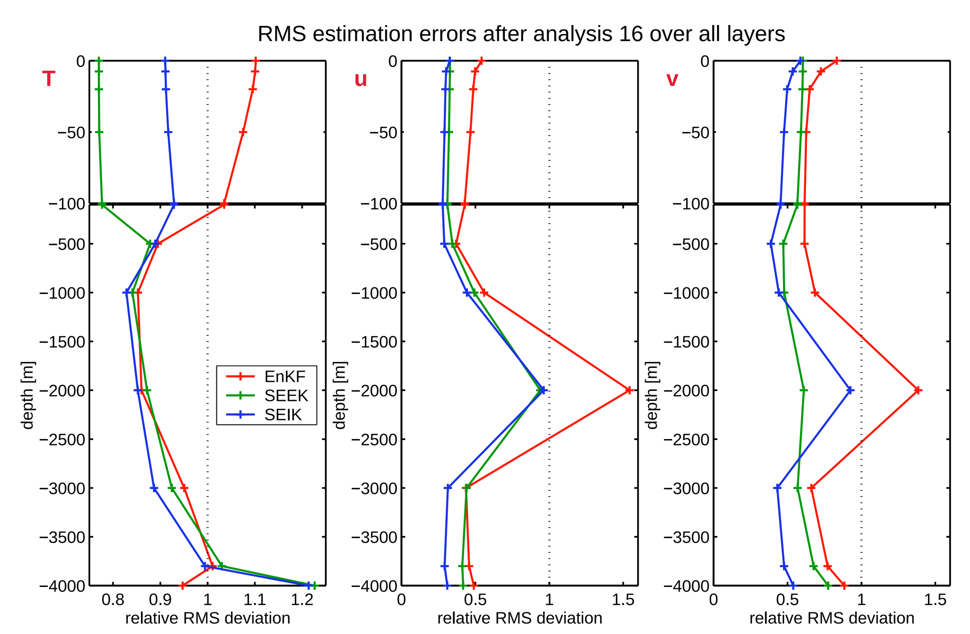
Conclusions

- ⇒ For $N = 10$ the SEIK filter yields a significantly better filter performance than the EnKF. For large ensemble sizes the performance of the EnKF and SEIK algorithms converge.
- ⇒ The SEEK filter behaves distinct from the EnKF and SEIK filters. This is due to the different forecast scheme applied in SEEK.
- ⇒ Employing only observations of the sea surface height with a rather large error, the different model fields are well estimated for all three filters. The temperature fields show a larger estimation error - a limited number of temperature observations would enhance the estimation quality of temperature for all three filters.
- ⇒ Computing time for the assimilations is determined by the model forecasts. The SEIK filter shows the best filter performance for smallest ensemble sizes. Thus, it appears to be particularly suited for data assimilation with large scale models.

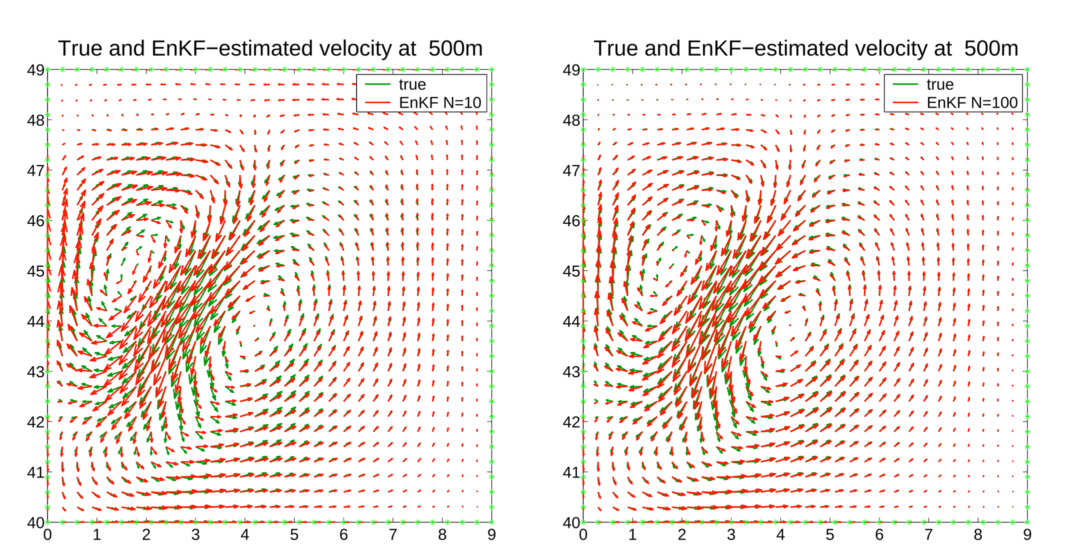
N = 100



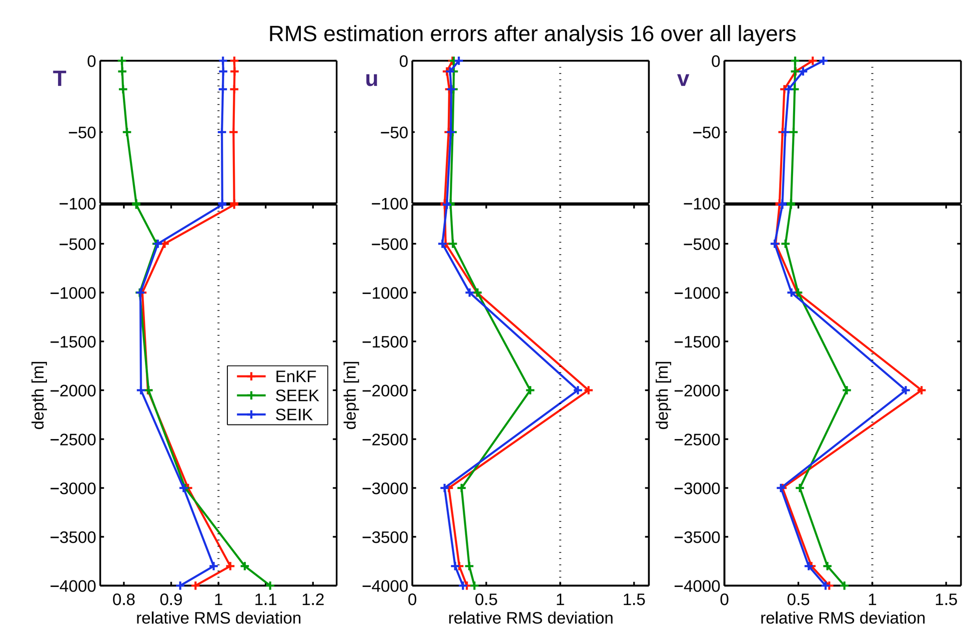
RMS estimation errors for the four model fields for assimilation with an ensemble size of $N = 100$. The RMS values are scaled by the RMS deviation of a simulation without assimilation from the true state fields.



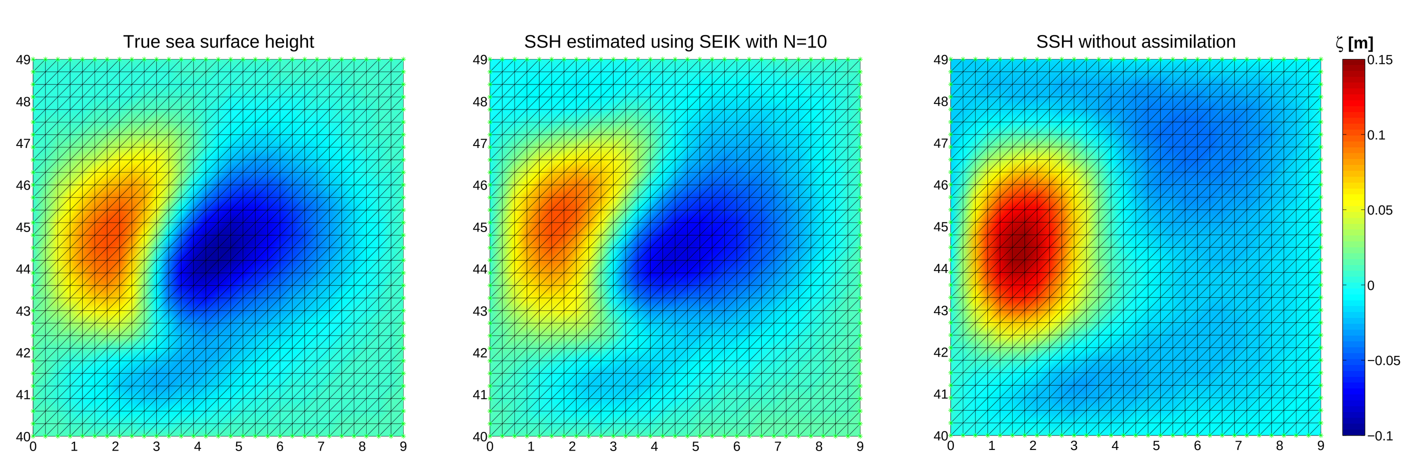
Relative RMS estimation errors at the end of the assimilation period displayed over all layers for $N = 10$.



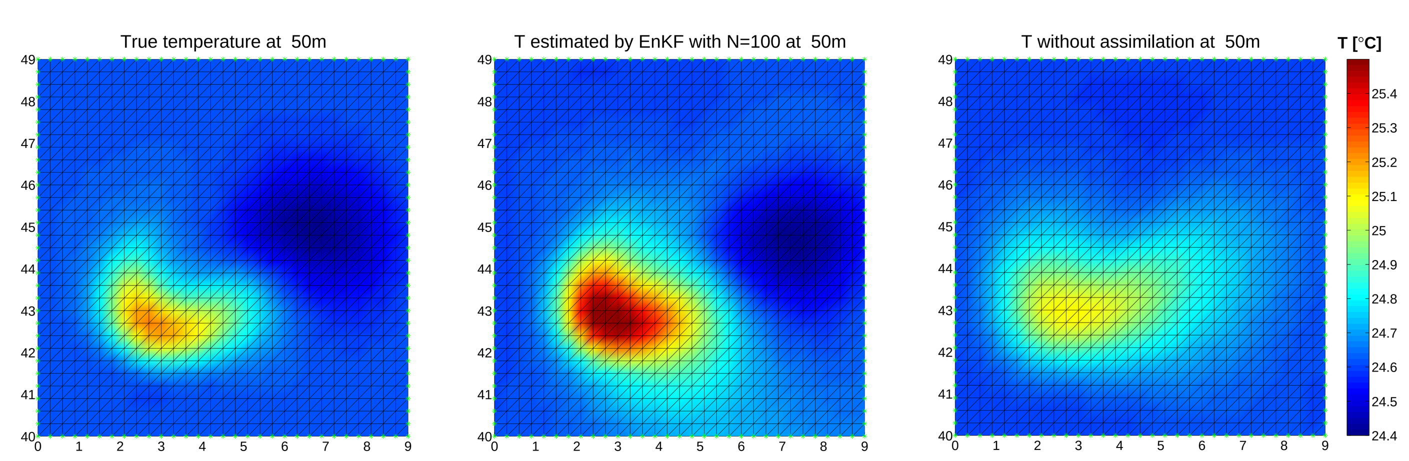
Comparison of true and estimated velocity fields at 500m depth at the end of the assimilation period. Show are the true velocity field and the velocity estimated using the EnKF with N=10 (left) and N=100 (right).



Relative RMS estimation errors at the end of the assimilation period displayed over all layers for $N = 100$.



Sea surface height ζ at the end of the assimilation period. Shown are (from left to right) the true ζ, that estimated by the SEIK filter with N=10, and ζ from a simulation initialized as the filter experiments, but without assimilation.



Temperature **T** at 50m depth at the end of the assimilation period. Shown are (from left to right) the true temperature field, that estimated by the EnKF filter with N=100, and **T** from a simulation initialized as the filters, but without assimilation.